WAR DEPARTMENT

TECHNICAL MANUAL

MATHEMATICS FOR AIR CREW TRAINEES

February 26, 1943

TECHNICAL MANUAL No. 1-900 } WAR DEPARTMENT, WASHINGTON, February 26, 1943.

MATHEMATICS FOR AIR CREW TRAINEES

SECTION I

GENERAL

1. Purpose and scope.—*a.* The purpose of this manual is to provide, in convenient form, a review of basic arithmetic and related subjects which members of an air crew must understand to practice simple air navigation and cope with other problems of the practical airman.

b. Briefly, the scope includes such elementary operations of arithmetic as addition, subtraction, multiplication, division, percentage, ratio and proportion, angular measurements, scales, and the use of graphs and formulas, together with the graphical solution of some of the more common problems involving the triangle of velocity.

c. In mathematics, as in learning to fly, no amount of reading can replace actual practice. For this reason, many exercises have been included in each paragraph, and at the end of each section a collection of miscellaneous exercises has been added based on the material considered in that section. It is not contemplated that every student will do all of the exercises. However, an ample number of exercises has been inserted to provide an opportunity for those trainees who may feel the need for extra practice. The answers to the even numbered exercises are given to enable each student to check his own work if he wishes. Illustrative examples are profuse and should help to clarify difficult points which may arise.

*This manual supersedes TM 1-900, April 22, 1942.

509469°—43——1

d. Undoubtedly, some of the topics will seem elementary to many of the students. It must be remembered, however, that the mathematical proficiency demanded of an airman not only involves an understanding of the various operations, but also the ability to perform these operations *accurately and quickly*, and often under trying circumstances. Therefore, the time spent in practicing such a simple operation as addition, for example, will give valuable training, no matter how clearly the process is understood.

2. Materials.—In addition to pencil and paper, the student will need a ruler, a protractor, and a few sheets of graph paper.

SECTION II

FUNDAMENTAL OPERATIONS

Paragraph

3. Purpose and scope.—The purpose of this section is to provide a review of the four fundamental operations of arithmetic: addition, subtraction, multiplication, and division. Upon these fundamental operations all other mathematical calculations are based. One or more of them must be used in solving any problem.

4. Addition.—*a.* Addition is the operation of finding the sum of two or more numbers. To add several numbers, place the numbers in a vertical column so that the decimal points are all in a vertical line. (When no decimal point is indicated, it is assumed to be on the right.) Then add the figures in the right-hand column and place the sum under this column. If there is more than one figure in this sum, write down only the right-hand figure and carry the others to the next column to the left.

Example: Find the sum of 30.53, 6.475, 0.00035, and 3476.

Solution:

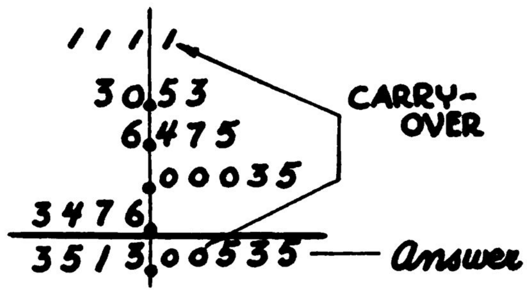


FIGURE 1.

b. Units.—Almost all the numbers which arise in practical arithmetic have to do with definite quantities such as 78 *feet,* 239 *miles,* 25 *degrees,* 160 *miles per hour,* 210 *pounds,* and so on. In these examples, the words in italics, which state what the quantities are in each case, are called the *units.* When adding several quantities together, it is clear that the units must all be the same. For example, "the sum of 78 *feet* and 160 *miles per hour*" is a completely meaningless statement.

(1) Units are so important and occur so often that standard abbreviations have been adopted for them. A list of the correct abbreviations and the relations which exist between some of the units are given in the appendix.

(2) *Example:* Find the sum of 78 feet and 3 miles.

Solution: In this case, since 1 mile is the same as 5,280 feet (see appendix), then 3 miles are the same as 15,840 feet. Therefore, 78 feet and 15,840 feet may be added together to give 15,918 feet. But the student is cautioned that unless there is a relation between the various units so that all the quantities may be expressed in terms of the *same* units, the addition *cannot* be performed.

c. Symbols.—In arithmetic and in other branches of mathematics, much space and effort are saved by using symbols. Thus, in order to write "find the sum of 70.765 and 301.4," the plus sign (+) is used and this phrase can be written simply as "70.765+301.4=?." When more than two numbers are to be added, the plus sign is repeated, for example: 70.765+301.4+765.84=1,138.005.

d. Exercises.

(1) 30.53 in.+6.475 in.=?

(2) 648.03 cm+37.895 cm+219.921 cm+.08376 cm=905.92976 cm *Answer.*

(3) 100.001+9.098+5678.91=?

(4) 897.1+0.989+900.76+91901.359=93700.208 *Answer.*

(5) 9876 ft+101.109 ft+77.007 ft+92.928 ft+94.987 ft+60.768 ft=?

(6) 19.767+43.542+76.305+58.143+13.25=211.007. *Answer.*

(7) 11.1111 miles+66.667 miles+1.222 miles+125.125 miles+375.375 miles=?

(8) 78.908+202.202+62.501+0.003594+75=418.614594 *Answer.*

(9) 7.8098+20.202+6.2501+000.3594+7.5=?

(10) 78.808 yd+98.15 yd+760 yd+88199.76 yd=89136.718 yd *Answer.*

5. Subtraction.—*a.* Subtraction is the operation of finding the difference between two numbers. In order to subtract one number from another, write the smaller number below the larger so that the decimal points are in a vertical column. Beginning with the right column, subtract the figures in the smaller number from the corresponding figures in the larger number above them.

Example: Subtract 765.3 from 986.7.

Solution:

```
 986.7
 765.3
 -----
 221.4
```

Answer.

b. If, however, the figure in the number being subtracted is larger than the figure directly above it, it is necessary to borrow one unit from the next figure to the left.

(1) *Example:* Subtract 765.3 from 843.1.

Solution:

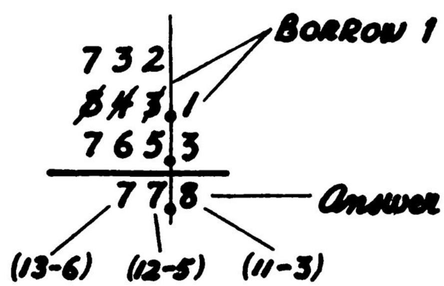


FIGURE 2.

(2) It is better to learn to do the "carrying over" mentally so that the preceding solution looks like this:

```
 843.1
 765.3
 -----
  77.8
```

Answer.

c. When a column has only one figure in it, zeros must be supplied in the blank spaces.

Example: Subtract 765.328 from 843.1.
Solution:

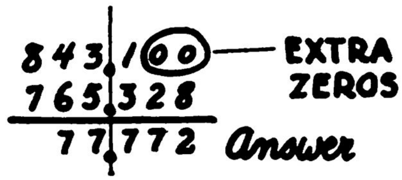


FIGURE 3.

d. A problem in subtraction may be checked by adding the answer to the number directly above it. The sum should *always* be the number in the top row.

(1) *Example:* Check the answer to the example in *c*(1) above.
Solution:

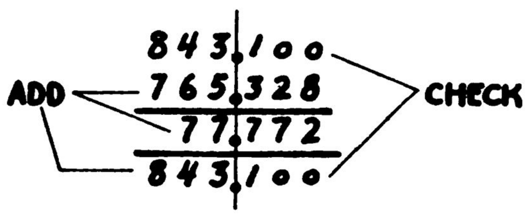


FIGURE 4.

(2) *Units.*—As in addition, care must be exercised to be sure that the units of the two quantities in a subtraction are the same.

e. Exercises.—In each of the following exercises subtract the smaller number from the larger. The symbol used to indicate subtraction is the *minus sign* (−). Therefore, an expression such as "1,205.789−980.833=?" means "What is the remainder when 980.833 is subtracted *from* 1,205.789?" The number following the minus sign is always subtracted from the number preceding the sign.

(1) 1,205.789 in.−980.833 in.=?
(2) 19.52−.78=18.74 *Answer.*
(3) 760,591−674,892=?
(4) 73.44 cu. in.−8.7375 cu. in.=64.7025 cu. in. *Answer.*
(5) 89.73−10.0045=?
(6) 941.7−87.372=854.328 *Answer.*
(7) 1,004.78 miles−1,004.164 miles=?
(8) 100,433 sq. ft−99,857 sq. ft=576 sq. ft *Answer.*
(9) 1,000,000.3−998,757.4=?
(10) 3,756.04−2,489.7=1,266.34 *Answer.*

6. Multiplication.—*a.* Multiplication is a short method of adding a number to itself as many times as may be indicated. The numbers multiplied together are called the *factors* and the result of the multiplication is called the *product.* To multiply two numbers together, or in other words, to find the product of two factors, first write the factors one below the other (see example below). It is usually easier to operate with the smaller number of figures in the bottom row. Multiply the factor in the top row by the right-hand figure of the factor in the bottom row, and write this partial product directly under the second factor. If there is more than one figure in the product the same "carrying over" procedure is followed as in addition. Then multiply the factor in the top row by the second figure from the right in the second factor, and write this second partial product so that its right-hand figure is directly under the figure that was used to find it. These partial products are then added together to yield the required product.

Example: Multiply 1,653 by 247.

Solution:

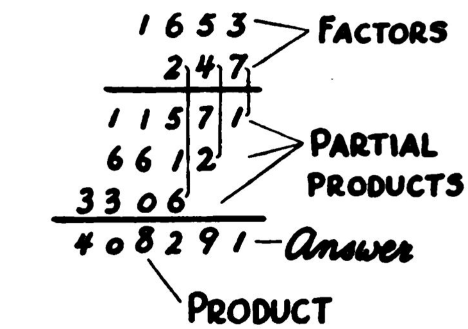


FIGURE 5.

b. When there are decimal points, they are ignored until the product has been found. Then the decimal point is inserted in the product according to the following rule: Count off the number of figures to the right of the decimal point in each factor. Then the number of figures to the right of the decimal point in the product is equal to the *sum* of the number of figures after the decimal point in each factor.

Example: Multiply 16.53 by 24.7.

Solution:

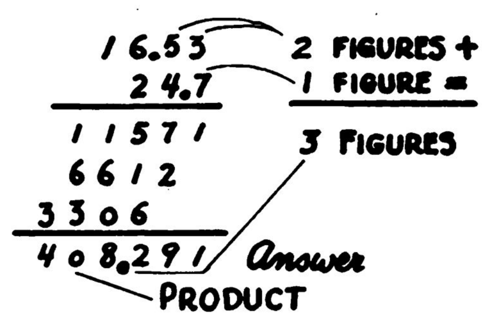


FIGURE 6.

c. When the lower factor contains zeros, the partial products corresponding to these zeros need not all be written down. Only the right-hand zero is written down. However, care must be exercised to have the right-hand figures of all the partial products directly below their corresponding figures in the second factor.

Example: Multiply 16.53 by 24.07.

Solution:

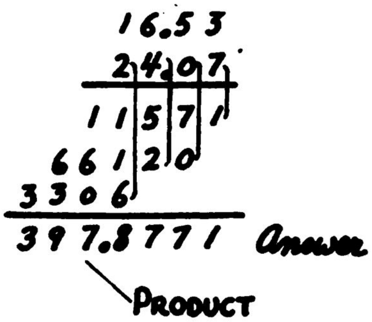


FIGURE 7.

d. Although there is no very simple way to check a multiplication, it is good practice to anticipate the approximate size of the product before beginning a long multiplication. This is done by "rounding off" the factors to permit easy mental multiplication. Although by no means an accurate check, this will frequently catch mistakes in addition or in the location of the decimal point which would otherwise result in nonsensical answers.

Example: What is the approximate product of 15.73 multiplied by 187.04?

Solution: Round off 15.73 to 15, and 187.04 to 200. Then the product is roughly 15 by 200=3,000. It is clear then that the product of 15.73 and 187.04 cannot be 150.6 or 6,030.3745, for example.

e. *Symbols and units.*—The more common symbol for multiplication is $\times$. However, it is quite common simply to write the numbers in parentheses next to each other: $(3.04)\ (17.78)=3.04\times 17.78=54.0512$, for example.

(1) When the same number is to be multiplied by itself, for example 3.04×3.04, this is usually indicated by a small "2" placed above and to the right of the number: $3.04\times 3.04=3.04^2$. This is read as "3.04 *squared*," and 3.04^2 is the "*square* of 3.04." If the number is to be used as a factor 3 times, then a small "3" is used: $3.04\times 3.04\times 3.04=3.04^3$. This is read as "3.04 *cubed*," and 3.04^3 is the "*cube* of 3.04."

(2) Unlike addition and subtraction, multiplication of different units can be performed. The product is then expressed in a unit which is itself the product of the units of the factors.

(*a*) *Example:* Multiply 5 pounds by 7 feet.

Solution: (5 pounds) (7 feet)=35 pounds-ft=35 lb-ft=35 ft-lb *Answer.*

(*b*) *Example:* Multiply 9 feet by 17 feet.

Solution: (9 feet) (17 feet)=153 feet$\times$feet=153 $(\text{feet})^2$=153 ft^2 (ft^2 is read "square feet") *Answer.*

f. In arithmetic, as in other operations, there are many tricks which often simplify the work. One such trick which is useful and easy to remember is the following: To multiply any number by 25, move over the decimal point in the given number two places to the right; then divide by 4.

Example: Multiply 16.53 by 25.

Solution: Moving the decimal point over two places, 16.53 becomes 1653. Dividing this by 4: 1653/4=413.25. Therefore 16.53×25=413.25 *Answer.*

g. Exercises.—To each of the following exercises three answers have been given. Eliminate the answers which are obviously wrong by rounding off the factors and finding the approximately correct answers mentally.

(1) $600.3\times 42.7=\left\{\begin{array}{l}25{,}632.81\\ 1{,}200.62\\ 4{,}273.21\end{array}\right.$

(2) $180\times 76=\left\{\begin{array}{l}30{,}740\\ 13{,}680\\ 25{,}580\end{array}\right.$

(3) $12.45\times 18.3=\left\{\begin{array}{l}400.785\\ 60.785\\ 227.835\end{array}\right.$

(4) $88\times 3.2=\left\{\begin{array}{l}472.6\\ 281.6\\ 31.7\end{array}\right.$

(5) $1{,}751.2\times 36.4=\left\{\begin{array}{l}63{,}743.68\\ 6{,}374.368\\ 12{,}743.68\end{array}\right.$

(6) $903\times 8.475=\left\{\begin{array}{l}3{,}652.925\\ 7{,}652.925\\ 76.52925\end{array}\right.$

h. Exercises.—Perform the indicated multiplications:

(1) .0734 in.×70.34 in.=?

(2) 831.43×71.46=59,413.9878 *Answer.*

(3) 1.0073 in.×6.4 ft=?

(4) 8.94×9.37=83.7678 *Answer.*

(5) 8,374.5×9,378.46=?

(6) 10,742 lb×737.2 ft=7,919,002.4 ft-lb *Answer.*

(7) 23.1×847.4=?

(8) 9,034×10.06=90,882.04 *Answer.*

(9) 8.037 ft×24.2 lb=?

(10) .0074×371.5=2.7491 *Answer.*

7. Division.—*a.* Division is the process of finding how many times one number is contained in another. The number to be divided is called the *dividend*, and the number by which it is divided is called the *divisor*. The result of the operation, or the answer, is called the *quotient*.

b. When the divisor contains but one figure, the method commonly used is known as *short division*. To perform short division, place the divisor (one figure) to the left of the dividend, separated by a vertical line (see example below). Then place a horizontal line over the dividend. Divide the first or the first two figures of the dividend, as is necessary, by the divisor and place the quotient over the line. If the divisor does not go an even number of times, the remainder is prefixed to the next figure in the dividend and the process is repeated.

Example: Divide 4,644 by 6.

Solution:

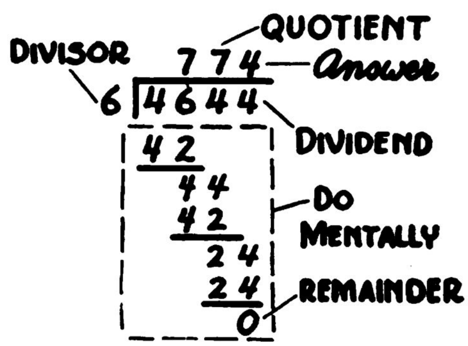


FIGURE 8.

c. When the divisor contains two or more figures, the method used is known as *long division*. This is performed as follows: Place the divisor to the left of the dividend, separated by a line, and place the quotient above the dividend, as in short division. Using the divisor, divide the first group of figures of the dividend which gives a number as large or larger than the divisor (see example below). Place the first figure of the quotient above the dividend. Then multiply this figure by the divisor, and place the product below the figures of the dividend which were used for this division. Then subtract this product from the figures directly above it. The next figure in the original dividend is brought down to form a new dividend. This is repeated until all the figures of the original dividend have been used.

Example: Divide 6,646,250 by 10,634.
Solution:

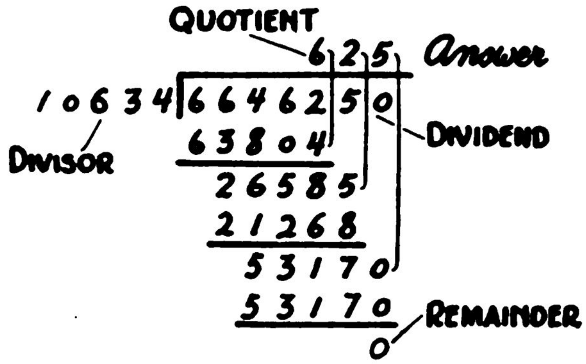


FIGURE 9.

d. It is not very common in either short or long division to have the divisor go into the last trial dividend a whole number of times. When the last trial remainder is not zero, it must be indicated in the answer.

(1) *Example:* Divide 4,647 by 6.
Solution:

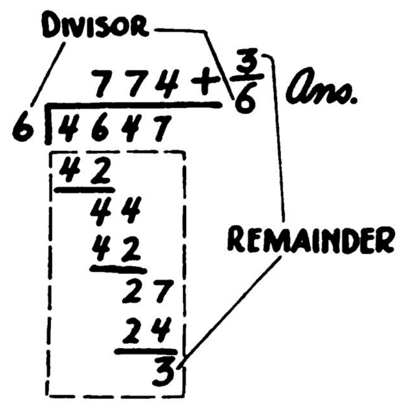


FIGURE 10.

(2) *Example:* Divide 6,646,257 by 10,634.
Solution:

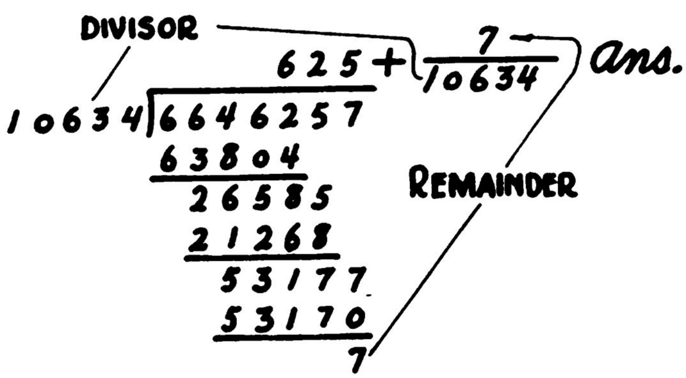


FIGURE 11.

e. Symbols.—"4,647 divided by 6" may be indicated in symbols in several ways. The division sign may be used: 4,647÷6. The "stroke" is more convenient to use on the typewriter: 4,647/6. Finally, the division may be indicated as a *fraction:* $\frac{4647}{6}$. The fact that 4,647 divided by 6 is 774 with a remainder of 3 may be written as "$4647 \div 6 = 774 + \frac{3}{6}$" or "$4647/6 = 774 + \frac{3}{6}$" or "$\frac{4647}{6} = 774 + \frac{3}{6}$."

f. Decimal point.—To locate the decimal point in the quotient when decimal points are present in either the divisor or the dividend, move the decimal point in the divisor to the right of the right-hand figure. Then move the decimal point in the dividend to the right the same number of places that the point was moved in the divisor. When dividing, be careful to place the quotient so that each figure of the quotient is directly above the right-hand figure of the group of figures which were used in the dividend. Then the decimal point in the quotient will be directly above the new position of the decimal point in the dividend. It will also be helpful to remember that the number of decimal places in the quotient is equal to the difference between the number of decimal places in the dividend and divisor.

(1) *Example:* Divide 4.644 by .06.
Solution:

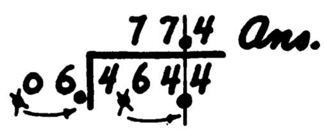


FIGURE 12.

(2) *Example:* Divide 6.646250 by 10.634.
Solution:

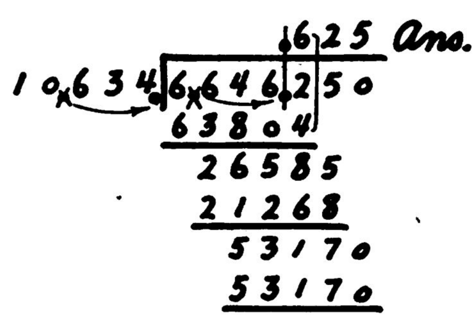


FIGURE 13.

g. If decimal points are involved in a division, as a rule the remainder is not indicated when the division does not come out even. In lieu thereof extra zeros are added to the dividend, and the division is continued until the quotient has as many figures as desired.

(1) *Example:* Find 46.47/0.6.
Solution:

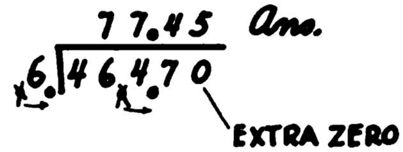


FIGURE 14.

(2) *Example:* Find 6.646250÷10.637.
Solution:

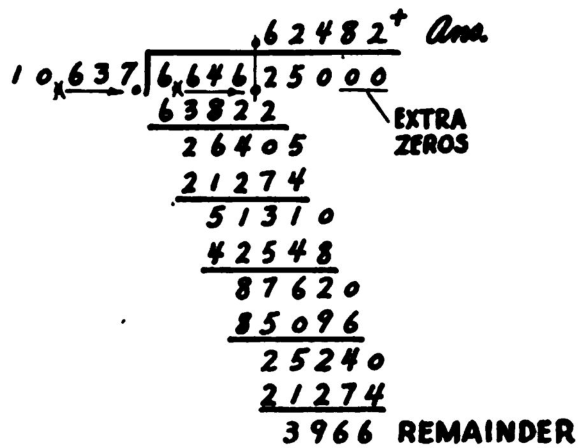


FIGURE 15.

h. Mixed numbers.—In *e* above it was stated that "$\frac{4647}{6}=774+\frac{3}{6}$." When the plus (+) sign is omitted, then 774$\frac{3}{6}$ is called a *mixed number*. A mixed number is simply the sum of a whole number and a fraction written without the plus sign. To convert a mixed number to a pure fraction, multiply the whole number by the denominator of the fractional part and add the numerator. This is the new numerator; the denominator does not change.

Example: Change 78$\frac{4}{5}$ to a pure fraction.
Solution:

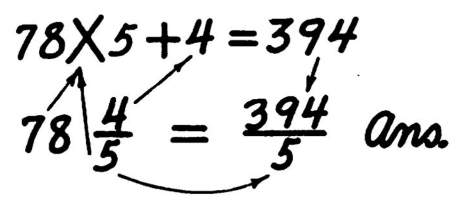


FIGURE 16.

i. Checks.—Any division may be checked by multiplying the divisor by the quotient and adding the remainder. The result is *always* the dividend.

Example: Check the answer to example in $g(2)$ above.

Solution: $.62482 \times 10.637 = 6.64621034$

$$6.64621034 + \text{remainder} = 6.64621034$$
$$\underline{.00003966}$$
$$6.64625000 \qquad \textit{Check.}$$

j. Units.—As in multiplication, any two different quantities may be divided, even though the units are not the same. The quotient is expressed in a unit which is itself the quotient of the units of the dividend and the divisor.

(1) *Example:* Divide 175 miles by 10 hours.

Solution: $\frac{175 \text{ miles}}{10 \text{ hours}} = 17.5 \frac{\text{miles}}{\text{hours}}$ *Answer.*

(In units of this type it is customary to write the denominator in the singular and to use the stroke (/) to separate the numerator from the denominator: 17.5 miles/hour, or 17.5 miles/hr. Although "miles/hour" really means miles divided by hours it is usual to substitute the word "per" for "divided." Hence "miles/hour" is read as miles *per* hour, the standard abbreviation for which is mph.

(2) *Example:* Divide 500 pounds by 50 square inches.

Solution: $\frac{500 \text{ pounds}}{50 \text{ square inches}} = 10$ pounds/square inch $= 10$ lb/sq. in.

Answer.

(10 pounds per square inch)

k. Exercises.—Perform the indicated divisions and express the quotient as a mixed number.

(1) 894/16

(2) 755/24

Solution:
```
     31
24/755
   72
    35
    24
    11
```
$31\frac{11}{24}$ *Answer.*

(3) 1,025/314

(4) 215/72 $2\frac{71}{72}$ *Answer.*

(5) 4,723/353

(6) 6,754,000/11,411 $591\frac{10,099}{11,411}$ *Answer.*

(7) 9,001/30

(8) 11,415/45 $253\frac{2}{3}$ *Answer.*

(9) 673/37

(10) 1,371/38 $36\frac{3}{38}$ *Answer.*

l. Exercises.—In the following exercises, express the quotient as a decimal. Round off the decimal part of the quotient to two places.

(1) 73.01/3.4

(2) .345/.36

```
                  .958
Solution: 36/34.500
             32 4
              2 10
              1 80
                 300
                 288
```

Since only two decimal places are to be obtained, the quotient is rounded off to .96. *Answer.*

If the figure to be thrown away is greater than or equal to 5, increase the figure on the left by 1. If the figure is less than 5, do not change the preceding figure. Thus: .953=.95; 1.057=106; 1.053=1.05, etc.

(3) 13.37/.834

(4) 14.705/8.64 — 1.70 *Answer.*

(5) (157 miles)/(17.3 hours)

(6) 1,942.4/.0035 — 554971.43 *Answer.*

(7) 9.63/145.4

(8) (198 miles)/(59 minutes) — 3.19 miles/min. *Answer.*

(9) (5,280 feet)/(60 seconds)

(10) 19.437/38.6 — .50 *Answer.*

8. Conversion of decimal fractions to common fractions.—*a.* A number which consists of a decimal point followed by a sequence of figures is called a *decimal fraction.* Thus, .33, .9899, .00467, and .00335 are all decimal fractions. Since 33 divided by 100 is .33, then $.33=\frac{33}{100}$. Similarly, $.9899=\frac{9,899}{10,000}$, and $.00467=\frac{467}{100,000}$. Therefore, to express any decimal fraction in fractional form, write the number without the decimal point and divide it by 1 followed by as many zeros as there are figures after the decimal point in the given number.

(1) *Example:* Express .023678 in fractional form.

Solution:

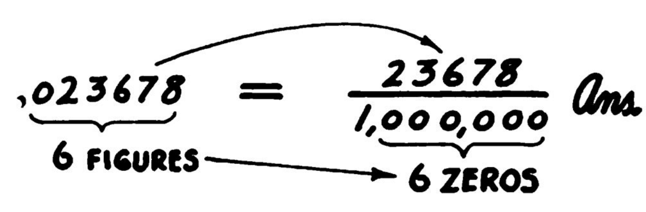


FIGURE 17.

(2) *Example:* Express 4.0785 in fractional form.
Solution: First express .0785 as a fraction:

$$.0785=\frac{785}{10,000}$$

$$4.0785=4+\frac{785}{10,000}=4\ \frac{785}{10,000} \qquad \textit{Answer.}$$

b. As stated in paragraph 7*e*, a fraction such as 54/16 or $\frac{54}{16}$ is really just an indicated division. Very frequently in calculations it is much easier to carry a fraction along as a fraction than it is to "divide it out." Later on, this problem will be considered in detail. At present, however, there is one very important rule of operation on fractions which should be mastered.

(1) This rule is that both the dividend (numerator) and divisor (denominator) of any fraction may be divided or multiplied by any number (except zero), without changing the value of the fraction. For example, if the numerator (54) and the denominator (16) of the fraction $\frac{54}{16}$ are both divided by 2, then according to this rule $\frac{54}{16}=\frac{27}{8}$.

(2) This rule allows zeros to be added to any given decimal number. Since $.27=\frac{27}{100}$, for example, by the rule, both numerator and denominator can be multiplied by 10. Then $\frac{270}{1,000}=.270$ and because the fractional value is unchanged, .27=.270.

c. A fraction is said to be in its lowest terms or simplest form if there is no number which will divide both the numerator and denominator evenly. The operation of finding the simplest form of a fraction is called *reduction* to lowest terms or *simplification.*

Example: Simplify $\frac{632}{32}$

Solution: Divide numerator and denominator by 4:

$$\frac{632}{32}=\frac{158}{8}$$

This can be simplified still more by dividing by 2:

$$\frac{158}{8}=\frac{79}{4} \qquad \textit{Answer.}$$

d. Exercises.—In the following exercises express the given decimal fractions in fractional form and then simplify. When possible, express the simplified fraction as a mixed number.

(1) 1.875=?

(2) .9375=? $\frac{15}{16}$ *Answer.*

(3) 2.109375=?

(4) .125=? $\frac{1}{8}$ *Answer.*

(5) .890625=?

(6) 3.828125=? $3\frac{53}{64}$ *Answer.*

(7) .625=?

(8) 4.375=? $4\frac{3}{8}$ *Answer.*

(9) 1.6875=?

(10) 2.3125=? $2\frac{5}{16}$ *Answer.*

e. Percentage.—Percent means a number with an understood denominator of 100. For example, 50 percent (%) means $\frac{50}{100}$ or .50.

(1) To change from percent to a decimal, divide the number of percent by 100, which is equivalent to moving the decimal point two places to the left, and omit "percent."

(*a*) *Example:* Change 42 percent to a decimal.
Solution: 42 percent=42/100=.42 *Answer.*

(*b*) *Example:* Change .9 percent to a decimal.
Solution: .9 percent=.9/100=.009 *Answer.*

(*c*) *Example:* Change $\frac{1}{2}$% to a decimal.

Solution: $\frac{1}{2}$%=.5%=.5/100=.005 *Answer.*

(2) To change from a decimal to a percent, multiply the decimal fraction by 100, which is equivalent to moving the decimal point two places to the right, and add "percent" (or "%").

(*a*) *Example:* Express .45 as a percent.
Solution: .45×100=45 percent *Answer.*

(*b*) *Example:* Express 6.47 as a percent.
Solution: 6.47×100=647 percent *Answer.*

(*c*) *Example:* Express .0048 as a percent.
Solution: .0048×100=.48% *Answer.*

(3) It is common to specify changes in troop strength, population, enlistments, and even changes in the physical dimensions of pistons, for example, in terms of percent. The following examples illustrate the methods of solving this type of problem:

(*a*) *Example:* If the population of the United States was 120,000,000 in 1930 and increased 7 percent during that year, what was it in 1931?

Solution: 7 percent=.07
.07×120,000,000=8,400,000.

Therefore the population in 1931 was 120,000,000+8,400,000=128,-400,000 *Answer.*

(*b*) *Example:* At a certain altitude and temperature, the true air speed is 14 percent greater than the calibrated air speed. If the calibrated air speed is 172 mph, what is the true air speed?

First solution: 14 percent=.14

.14×172 mph=24 mph

Therefore 172 mph+24 mph=196 mph *Answer.*

Second solution: 114 percent=1.14

1.14×172 mph=196 mph *Answer.*

(*c*) *Example:* At a certain altitude and temperature, the true air speed is 14 percent greater than the calibrated air speed. If the true air speed is 210 mph, what is the calibrated air speed?

Solution: Caution must be used in this type of problem. Percent increases as usually understood mean so many percent of the *smaller* number. Consequently, since 14 percent=.14,

calibrated air speed+(.14)×(calibrated air speed)=210 mph

Therefore (1.14) (calibrated air speed)=210 mph

or calibrated air speed=(210 mph)/1.14

=184 mph *Answer.*

This kind of problem can be checked very easily. Since 14 percent of calibrated air speed=.14×184=26 mph, 14 percent of calibrated air speed+calibrated air speed=26 mph+184 mph=210 mph

Check.

(4) To change a common fraction to a percent, first change the fraction to a decimal and then change the decimal to a percent.

(*a*) *Example:* Express $\frac{3}{4}$ as a percent.

Solution: $\frac{3}{4}=.75=75\%$.

(*b*) *Example:* Express $\frac{1}{3}$ as a percent.

Solution: $\frac{1}{3}=.333\ldots=33.33\ldots\%$ or $33\frac{1}{3}\%$ *Answer.*

(5) All types of percentage problems may easily be solved by the "equation" method.

(*a*) *Example:* What is 65% of 220?

Solution: x is 65% of 220

$x=(.65)(220)$

$x=143$ *Answer.*

NOTE.—"Is" always stands for "equals." "Of" always stands for "times." In the equation the percent must be used in the decimal form.

(*b*) *Example:* 250 is what percent of 900?

Solution: $250=x\cdot 900$

$x=\frac{250}{900}=.28=28\%$ *Answer.*

(*c*) *Example:* 65 is 13% of what number?

Solution: $65 = .13x$

$$x = \frac{65}{.13} = 500$$ *Answer.*

NOTE.—To solve these equations it is always necessary to divide by the multiplier of x.

f. *Exercises.*—(1) If 23 percent of a class of 1,900 cadets attend two engine training schools, find the number of cadets in this class who attended these schools.

(2) If 36 percent of a class of 1,800 cadets take their advanced training in Alabama, how many cadets does this represent? 648 *Answer.*

(3) A lieutenant calls a detail of 24 men. This represents 6 percent of the men in his squadron. How many men are there in the squadron?

(4) At 1,000 rpm, a propeller uses 80 percent of the horsepower developed. If the engine develops 1,500 horsepower at 1,000 rpm, how many horsepower are used by the propeller? 1,200 hp *Answer.*

(5) The top speed of an aircraft at 8,000 feet is 314 mph. At 12,000 feet the top speed has increased 8 percent. What is the top speed of the aircraft at 12,000 feet?

(6) For a certain air density, the true altitude is 16 percent greater than the calibrated altitude. If the true altitude is 17,400 feet, find the calibrated altitude. 15,000 ft *Answer.*

(7) At 15,000 feet altitude and at −10° C., the calibrated air speed is 240 mph. The true air speed is 15 percent more than the calibrated air speed. Find the true air speed.

(8) A sample of nickel steel contained 25.61 percent of nickel and 0.17 percent of carbon. How much nickel and how much carbon would be found in a ton of this nickel steel? 512.2 lb nickel *Answer.*
3.4 lb carbon

(9) A machine shop employing 225 men is forced to employ 36 percent more men. What is the increase in the number of employees?

(10) If the percent of direct hits out of a bomb load is estimated at 35%, and if 7 direct hits are needed to destroy an objective, how many bombs must be dropped? 20 bombs *Answer.*

9. Conversion of common fractions to decimal fractions.—*a.* To convert any fraction to a decimal form, simply add a decimal point on the right of the numerator and perform the indicated division as in paragraph 7*g*.

(1) *Example:* Express $\frac{5}{8}$ as a decimal fraction.

Solution: $8\overline{)5.0000}$ = .6250   Therefore $\frac{5}{8} = .625$ *Answer.*

(2) *Example:* Express $\frac{19}{4}$ as a decimal.

Solution: $4\overline{)19.000000}$ = 4.7500 Therefore $\frac{19}{4}=4.75$ *Answer.*

b. To express any fraction as a percent, first change the fraction to a decimal fraction as in *a* above, then change the decimal fraction to a percent as in paragraph 8.

Example: Express $\frac{7}{12}$ as a percent.

Solution: $12\overline{)7.000000}$ = .583333 . . . Therefore $\frac{7}{12}=$ .583333 . . .

(The series of dots indicates that the decimal fraction may be continued indefinitely by adding 3's.)

Therefore $\frac{7}{12}=.583333\ldots=58.3$ percent *Answer.*

c. Percent problems.—The following examples illustrate the converse (reverse) of those given in paragraph 8.

(1) *Example:* If 18 airplanes out of a squadron of 27 airplanes are available for combat, what percent of the squadron aircraft is available for combat? What percent is not available for combat?

Solution: $\frac{18}{27}=.6666\ldots=67$ percent available for combat *Answer.*

100 percent−67 percent=33 percent not available for combat

or $\frac{9}{27}=.333\ldots=33$ percent *Answer.*

(2) *Example:* At a pressure altitude of 20,000 feet with the air temperature at 10° below zero, the calibrated air speed is 200 knots, and the true air speed is 282 knots. What is the percent increase in the air speeds? Compare the true air speed with the calibrated air speed.

Solution: 282 knots−200 knots=82 knots.
82 knots/200 knots=.41=41 percent increase *Answer.*
282 knots/200 knots=1.41=141 percent. Therefore the true air speed is 141 percent of the calibrated air speed *Answer.*

d. Exercises.

(1) The top speed of an aircraft at 8,000 feet is 304 mph. At 12,000 feet the top speed is 352 mph. What is the percent of increase in speed?

(2) The indicated horsepower of an engine is 1,500, while the actual effective horsepower is 1,275. What percent of the indicated horsepower is the actual horsepower? 85 percent *Answer.*

Express each of the following as a decimal fraction:

(3) $\frac{7}{8}$

(4) $\frac{5}{12}$ .4166 . . . *Answer.*

(5) $\frac{9}{16}$

(6) $\frac{3}{32}$ .09375 *Answer.*

Express each of the following as a percent:

(7) $\frac{1}{4}$

(8) $\frac{3}{8}$ 37.5 percent *Answer.*

(9) $\frac{2}{5}$

(10) $\frac{7}{16}$ 43.75 percent *Answer.*

10. Addition and subtraction of fractions.—*a.* When several fractions which all have the same denominator are to be added the addition is performed by simply adding the numerators.

Example: $\frac{5}{7}+\frac{8}{7}+\frac{3}{7}+\frac{10}{7}=?$

Solution: $\frac{5}{7}+\frac{8}{7}+\frac{3}{7}+\frac{10}{7}=\frac{5+8+3+10}{7}=\frac{26}{7}$ *Answer.*

b. When one fraction is to be subtracted from another, and both have the same denominator, the subtraction is performed by simply subtracting the numerators.

Example: $\frac{9}{11}-\frac{6}{11}=?$

Solution: $\frac{9}{11}-\frac{6}{11}=\frac{9-6}{11}=\frac{3}{11}$ *Answer.*

c. When fractions to be added or subtracted do not all have the same denominator, the fractions must first be reduced to the same common denominator. What the common denominator is does not matter very much. The reduction to the common denominator is performed by applying the rule given in paragraph 8*b.*

(1) *Example:* $\frac{9}{11}+\frac{7}{10}+\frac{4}{5}=?$

Solution: By inspection it is found that all the fractions can be reduced to a denominator=110. The first is multiplied by 10 (both numerator and denominator), the second by 11, and the third by 22.

Then $\frac{9\times10}{11\times10}=\frac{90}{110}$; $\frac{7\times11}{10\times11}=\frac{77}{110}$; and $\frac{4\times22}{5\times22}=\frac{88}{110}$

therefore $\frac{9}{11}+\frac{7}{10}+\frac{4}{5}=\frac{90}{110}+\frac{77}{110}+\frac{88}{110}=\frac{90+77+88}{110}=\frac{255}{110}$

but $\frac{255}{110}=\frac{255/5}{110/5}=\frac{51}{22}$ *Answer.*

(2) The easiest way to find a common denominator is to multiply all the different denominators together. This will frequently give a common denominator which is larger than necessary, but if the answer is then reduced to lowest terms this method will always work.

(3) *Example:* $\frac{19}{25}-\frac{16}{33}=?$

Solution: $25\times33=825$

Then $\frac{19}{25}-\frac{16}{33}=\frac{627}{825}-\frac{400}{825}=\frac{627-400}{825}=\frac{227}{825}$ *Answer.*

d. Exercises:

(1) $\frac{2}{3}-\frac{1}{3}=?$

(2) $\frac{7}{8}-\frac{3}{4}=\frac{1}{8}$ *Answer.*

(3) $\frac{11}{16}-\frac{1}{2}=?$

(4) $\frac{2}{3}+\frac{3}{4}+\frac{5}{6}+\frac{7}{8}+\frac{11}{12}+\frac{13}{12}=5\frac{1}{8}$ *Answer.*

(5) $9\frac{1}{6}+8\frac{5}{3}+7=?$

Hint: Add $\frac{1}{6}$ and $\frac{5}{3}$ first.

(6) $7\frac{2}{3}+9\frac{3}{4}+11\frac{1}{2}=28\frac{11}{12}$ *Answer.*

(7) $14\frac{3}{4}+30\frac{1}{2}+4=?$

(8) A dealer had 16 gallons of oil to sell. He sold 1½ gallons to one customer, 2¾ gallons to another, 7¼ gallons to another, and the remainder to a fourth customer. How much did he sell to the fourth customer?

4½ gal. *Answer.*

(9) A man can do a piece of work in 13⅞ days. A boy can do the same piece of work in 19½ days. How much longer does it take the boy to do the work?

(10) The distance from outside to outside between two holes in a

steel plate is $6\frac{3}{8}$ inches. If one hole is $1\frac{1}{6}$ inches in diameter and the other is $2\frac{1}{24}$ inches in diameter, find the length of metal between the holes. $3\frac{11}{24}$ in. *Answer.*

11. Multiplication of fractions.—*a.* Two or more fractions are multiplied by multiplying the numerators together and multiplying the denominators together. The product is then a fraction the numerator of which is the product of the several numerators, and the denominator of which is the product of the denominators.

Example: Multiply $\frac{8}{9}$ by $\frac{17}{15}$.

Solution:

$$\frac{8}{9} \times \frac{17}{15} = \frac{8 \times 17}{9 \times 15} = \frac{136}{135} = 1\frac{1}{135} \text{ Ans.}$$

FIGURE 18.

b. To multiply mixed numbers, first change the mixed numbers to fractions, and then multiply as in *a* above.

Example: Multiply $15\frac{2}{3}$ by $19\frac{2}{5}$.

Solution: $15\frac{2}{3} = \frac{47}{3}$ and $19\frac{2}{5} = \frac{97}{5}$

$$\frac{47}{3} \times \frac{97}{5} = \frac{47 \times 97}{3 \times 5} = \frac{4559}{15} = 303\frac{14}{15}$$ *Answer.*

c. Exercises.

(1) $\frac{3}{4} \times 5 = ?$

(2) $\frac{6}{7} \times \frac{7}{6} = 1$ *Answer.*

(3) $12 \times \frac{3}{4} = ?$

(4) $63 \times 2\frac{2}{9} = ?$

Solution: $63 \times 2\frac{2}{9} = 63 \times \frac{20}{9} = \frac{\overset{7}{\cancel{63}}}{1} \times \frac{20}{\cancel{9}} = 140$ *Answer.*

(5) $12\frac{2}{3} \times 15\frac{2}{5} = ?$

(6) A tank holds 300 gallons of gas. If a pipe empties one-fourth of the gas in an hour, how many gallons will be left in the tank at the end of 2 hours?

Solution: $\frac{1}{4} \times 300 = 75$ gal./hr. In 2 hours, pipe will empty 150 gallons. Therefore $300 - 150$ gallons will be left. 150 gal. *Answer.*

(7) A tank is five-sixths full of gas. If one-eighth of this is drawn off, what part of the whole tank is drawn off? What part remains in the tank?

(8) The circumference of a circle is about $3\frac{1}{7}$ times the diameter. Find the circumference of a circle if the diameter is 14 feet; if 28 feet; if $\frac{1}{22}$ foot. 44 ft; 88 ft; $\frac{1}{7}$ ft *Answers.*

(9) If a motor makes 2,100 rpm, how many revolutions does it make in $\frac{3}{4}$ hour? In $3\frac{10}{21}$ days?

(10) An alloy used for bearings in machinery is $\frac{24}{29}$ copper, $\frac{4}{29}$ tin, and $\frac{1}{29}$ zinc. How many pounds of each in 246 pounds of the alloy?
286.34 lb copper; 47.72 lb tin; 11.93 lb zinc *Answers.*

12. Division of fractions.—*a.* To divide one fraction by another, first invert the divisor and then multiply as in paragraph 11.

Example: Divide $\frac{8}{9}$ by $\frac{15}{17}$.

Solution:

INVERT

$$\frac{8}{9} \div \frac{15}{17} = \frac{8}{9} \times \frac{17}{15} = \frac{136}{135} = 1\frac{1}{135} \text{ ans.}$$

FIGURE 19.

b. To divide mixed numbers, first change to a fractional form, then divide as in *a* above.

Example: Divide $7\frac{2}{3}$ by $8\frac{3}{4}$

Solution: $7\frac{2}{3} \div 8\frac{3}{4} = \frac{23}{3} \div \frac{35}{4} = \frac{23 \times 4}{3 \times 35} = \frac{92}{105}$ *Answer.*

c. Exercises.

(1) $\frac{5}{7} \div 10 = ?$

(2) $\frac{1}{3} \div \frac{4}{3} = ?$ $\frac{1}{4}$ *Answer.*

(3) $27\frac{3}{5} \div 9 = ?$

(4) $5\frac{1}{2} \div 2\frac{3}{4} = ?$ 2 *Answer.*

(5) In the blueprint of a house $\frac{1}{4}$ inch in the print represents 1 foot in the actual house. Find the dimensions of the rooms that measure as follows: $2\frac{1}{2}$ by $2\frac{1}{2}$ inches, $4\frac{1}{8}$ by $4\frac{5}{8}$ inches, $5\frac{3}{8}$ by 6 inches, $3\frac{1}{16}$ by $4\frac{19}{32}$ inches, respectively, on the blueprint.

(6) Two places, *A* and *B*, are 24 miles apart on a river that flows 3 miles an hour. A man can row 5 miles an hour in still water. He goes from *A* to *B* and back. Find the time for the journey.
Hint: Man's speed down river is 8 mph. 15 hr *Answer.*

(7) A car is going 11.25 miles per hour. How long will it take this car to go $468\frac{3}{4}$ miles?

(8) A layer of No. 8 wire, 0.162 inch in diameter, is wound on a pipe $24\frac{3}{8}$ inches long. How many turns of wire are wound on the pipe? 150.46 turns *Answer.*

(9) A mechanic can assemble 10/32 of a motor in 1 day. How many motors can he assemble in 3 3/8 days?

(10) If a pilot flies 347 miles in 3 hours, 15 minutes, how far will he travel at the same rate in 7 hours, 45 minutes?

827.46 miles. *Answer.*

13. Ratio and proportion.—*a*. Consider two bombs, one weighing 300 pounds and the other 100 pounds. The first is three times as heavy as the second, or the second is one-third as heavy as the first. This may be expressed as "the *ratio* of the weight of the second bomb to the weight of the first bomb is $\frac{1}{3}$." In other words, *a ratio is the quotient of two like quantities.* In this example,

$$\text{ratio} = \frac{100 \text{ lb}}{300 \text{ lb}} = \frac{1}{3}$$

b. The statement that two ratios are equal is called a *proportion.* Thus, for example, if the explosive in the first bomb is 270 pounds, and the explosive in the second bomb is 90 pounds, then the ratios of the explosives are also $\frac{1}{3}$, and

$$\frac{100 \text{ lb}}{300 \text{ lb}} = \frac{90 \text{ lb}}{270 \text{ lb}}$$

is called a proportion.

c. The utility of a proportion comes from the fact that if only one of the numbers is not known, it can easily be found. Suppose that two bombs are given, one weighing 450 pounds and the other weighing 150 pounds, and that the length of the first bomb is 36 inches. The length of bomb No. 2 is not known, but if the length of any bomb is "proportional" to its weight, then

$$\frac{\text{weight of bomb No. 1}}{\text{weight of bomb No. 2}} = \frac{\text{length of bomb No. 1}}{\text{length of bomb No. 2}}$$

is the proportion expressing this fact. Now some of these quantities are known:

$$\frac{450 \text{ lb}}{150 \text{ lb}} = \frac{36 \text{ in.}}{\text{length of bomb No. 2}}.$$

Therefore, if the proportion is true, then the length of bomb No. 2 must be 12 inches.

d. In mathematics, not only are symbols such as +, −, =, etc. used to simplify writing, but it is also convenient to introduce other symbols whenever they will shorten the work. Thus, to continue the preceding example, let

$$w_1 = \text{weight of bomb No. 1}$$
$$w_2 = \text{weight of bomb No. 2}$$

L_1=length of bomb No. 1
L_2=length of bomb No. 2

Then the proportion can be written even more simply as

$$\frac{w_1}{w_2}=\frac{L_1}{L_2} \text{ and } L_2=12 \text{ in.}$$

The practice of using letters to represent quantities is characteristic of all mathematics.

e. There is a general rule on proportions which can be stated very briefly in terms of symbols. Let a, b, c, and d be any quantities whatever. Then if there is a proportion between these quantities such that

$$\frac{a}{b}=\frac{c}{d}$$

then also $$ad=bc$$

That is, if the terms in a proportion be "cross-multiplied," the products are equal:

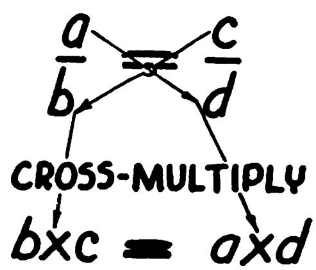


FIGURE 20.

Example: Let a=15 inches, b=60 inches, c=30 yards, and d=120 yards. Check the "cross-multiplication" rule for these quantities.

Solution: Since $\frac{15 \text{ in.}}{60 \text{ in.}}=\frac{30 \text{ yd}}{120 \text{ yd}}$, then

(15 in.) (120 yd) should equal (30 yd) (60 in.).
(15 in.) (120 yd)=1,800 in.-yd=(30 yd) (60 in.) *Check.*

f. The problem of converting from one type of unit to another can be regarded as a proportion problem.

Example: A speed of 12 miles per hour is equivalent to how many feet per second?

Solution: The known relation between the two types of units is that 1 mph=5,280/3,600 ft/sec.=22/15 ft/sec.

This gives the proportion

$$\frac{1 \text{ mph}}{12 \text{ mph}}=\frac{22/15 \text{ ft/sec.}}{X \text{ ft/sec.}}$$

Solving this for x gives $x=17.6$ ft/sec. Therefore, 12 mph is equivalent to 17.6 ft/sec. *Answer.*

g. Exercises.—It is customary to let x represent the unknown quantity. In the following exercises find the values of x which will make the proportions true.

(1) $\frac{5}{1}=\frac{x}{3}$

(2) $\frac{5}{6}=\frac{x}{3}$ in.

Solution: Cross-multiplying, $6x=15$ inches. If 6 x's are 15 inches, then obviously, one x must be one-sixth as much or 2.5 inches. In all of these problems, if the factor multiplying x is not 1, divide by the factor on both sides of the equality sign. Thus

$\frac{6x}{6}=\frac{15 \text{ in.}}{6}$ is the same as $6x=15$ inches. But $6x/6=x$.

Therefore $x=(15 \text{ in.})/6=2.5$ in. *Answer.*

(3) $\frac{5}{3}=\frac{x}{5}$

(4) $\frac{3}{7}=\frac{5}{x}$ $x=11⅔$ *Answer.*

(5) $\frac{.25}{.75}=\frac{3}{x}$

(6) $\frac{(¼)}{(.5)}=\frac{5}{x}$ $x=10$ *Answer.*

(7) $\frac{x}{y}=\frac{a}{b}$

(8) What is the value of A in the following proportions:

(*a*) $\frac{17}{45}=\frac{14}{A}$

(*b*) $\frac{3}{A}=\frac{10}{25}$ $A=7.5$ *Answer.*

(*c*) $\frac{\frac{3}{4}}{\frac{4}{5}}=\frac{6}{A}$

(9) Two pulleys are connected by a belt. The smaller one runs at a speed of 750 rpm and the larger at 200 rpm. What is the ratio of their speeds?

(10) An airplane travels 400 miles in 2 hours. Set up a proportion and determine how far the airplane will travel in 14 hours.

2,800 miles. *Answer.*

(11) If 1/10 inch on a map represents 49 miles, how many miles are represented by 3 inches on the map?

(12) If a boat drifts down stream 40 miles in 12 hours, how far will it drift in 15 hours? 50 miles. *Answer.*

(13) On June 12, 1939, a pilot flew a glider plane across Lake Michigan a total distance of 92 miles in 52 minutes. He cut loose from the tow plane at 13,000 feet and descended only 5,000 feet in crossing. At the same rate of descent, how much farther could he have glided? How many more minutes would he have been in the air?

(14) A roadbed rises 3⅓ feet in a horizontal distance of 300 feet. How many feet will the roadbed rise in 720 feet? 8 ft. *Answer.*

(15) If 16 gallons of gas will drive a car 288 miles, at the same rate of using gas how many gallons will it take to drive the same car from Chicago to Memphis, a distance of 564 miles?

h. Conversion exercises.—Obtain conversion factors required in the following examples from the appendix.

(1) Change 210 miles per hour to knots.

(2) How many feet per second are 32 miles per hour?

46.9 ft/sec. *Answer.*

(3) Express 58 centimeters in inches.

(4) Convert the following to nautical miles:

(*a*) 230 statute miles. 199.7 nautical miles. *Answer.*

(*b*) 34.5 statute miles. 29.9 nautical miles. *Answer.*

(*c*) 4,025 statute miles. 3,495.3 nautical miles. *Answer.*

(5) A tank containing 125 U. S. gallons of gas would contain how many British gallons?

(6) How many U. S. gallons are there in 78.5 British gallons?

94.2 U. S. gal. *Answer.*

14. Positive and negative numbers.—There are many quantities which by their contrary or opposite nature are best described as *negative* quantities in contrast to *positive* quantities. For example, temperatures above 0° Fahrenheit are considered as positive, whereas those below 0° are considered as negative. As a consequence it becomes necessary to consider negative and positive numbers and how to deal with them.

a. A negative number is indicated by prefixing a minus sign (—) in front of the number. Thus -5, -7.04, -90.003 are all negative numbers.

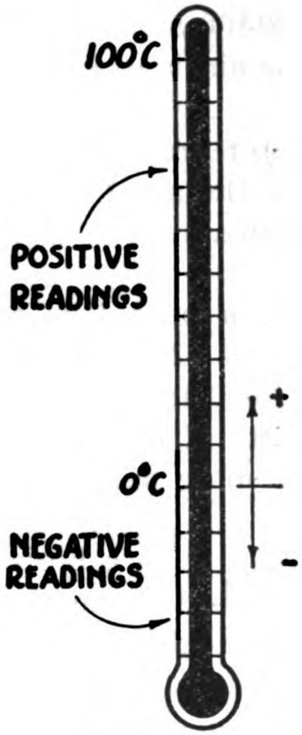


FIGURE 21.

b. A positive number is indicated by prefixing it with a plus sign (+), if necessary. When there is no possible ambiguity the plus sign is usually omitted. Thus +5, 7.04, +63.0, 98.4 are all positive numbers.

c. It is convenient to imagine the numbers as representing distances along a straight line as follows:

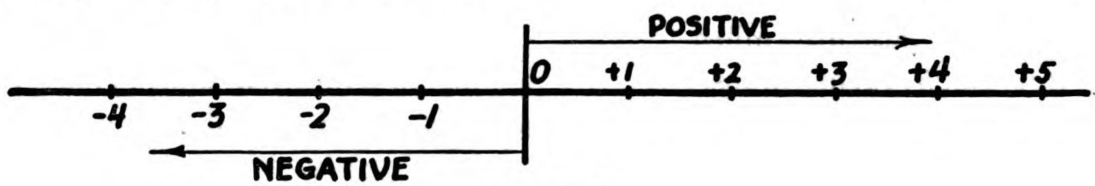


FIGURE 22.

Negative distances are measured to the left and positive distances are measured to the right.

d. The signs + and − now have additional meanings. They not only indicate addition and subtraction, but positive and negative numbers as well. To distinguish the sign of operation from the sign of quality (positive or negative), the quality sign is inclosed in parentheses: 25+(+5), 25−(+5), 25+(−5), or 25−(−5). For the sake of brevity, the first and second are generally written simply as 25+5, and 25−5.

15. Addition of positive and negative numbers.—To add two numbers which have the same signs, add the numbers and prefix the common (or same) sign. If the numbers to be added have unlike signs, find the difference and use the sign of the larger number.

a. Example: $(+6)+(-3)=?$

Solution: Since the signs are different, subtract the numbers to obtain a remainder of 3. Since the sign of the larger number is positive, the sign of the remainder is also positive: $6+(-3)=3$ *Answer.*

b. Example: $(-3)+(+2)=?$

Solution: Referring to figure 22, begin at -3 and count 2 units in a positive direction. The result is 1 space to the left of zero. Therefore $(-3)+(+2)=-1$.

This problem may also be done by using the rule stated in the preceding paragraph. Since the signs are unlike, subtract 2 from 3 and prefix the remainder by a minus sign since the larger number is negative. $(-3)+(+2)=-1$ *Answer.*

16. Subtraction of positive and negative numbers.—*a.* To subtract two numbers (positive or negative), change the sign of the number being subtracted and *add* the numbers as in addition (par. 15).

Example: $(-3)-(-4)=?$

Solution: Changing the sign of the number being subtracted, the problem then becomes—

$$(-3)+(+4)=+1$$ *Answer.*

b. Exercises.

(1) $(-6)+(+6)=?$

(2) $(+2)+(+12)=+14$ *Answer.*

(3) $(-62)+(-18)=?$

(4) $(+17)-(+15)=+2$ *Answer.*

(5) $(+32)-(-64)=?$

(6) $(-18)-(-64)=+46$ *Answer.*

(7) $(-17)-(-15)=?$

(8) $(-17.3)+(35.4)=+18.1$ *Answer.*

(9) $(-17.36)-(35.4)=?$

(10) $(-201.03)-(-10.4)=-190.63$ *Answer.*

(11) $-18+.4=?$

(12) $20-17.4+9=+11.6$ *Answer.*

(13) $-37.3+19.4+17.8=?$

(14) $(-175.03)+19=-156.03$ *Answer.*

17. Multiplication and division of positive and negative numbers.—*a.* If the two numbers to be multiplied have the same signs, then the product is positive. If the two numbers to be multiplied have opposite signs, then the product is negative.

(1) *Example:* Multiply (+3.04) by (17.8).
Solution: Since the signs are the same, the product is
+54.112 *Answer.*

(2) *Example:* $(+.00395)\times(-345.9)=?$
Solution: Since the signs are unlike, the product is negative, or
−1.366305 *Answer.*

b. Exercises.—Find the product in each of the following exercises.

(1) (−1.6) (.9)
(2) (−14.4) (−12) 172.8 *Answer.*
(3) (12.5) (1.25)
(4) (−9) (−8) (−6) −432 *Answer.*
(5) (−.17) (6) (−5)
(6) (2) (3.14) (9) 56.52 *Answer.*
(7) (−.4) (−.4) (−.4)
(8) $(-\frac{1}{3})$ $(\frac{9}{20})$ $(\frac{5}{6})$ $-\frac{1}{8}$ *Answer.*
(9) $(\frac{22}{7})$ (1.4) (1.4)
(10) $(-1\frac{2}{3})$ $(\frac{9}{10})$ $(-\frac{8}{15})$ $\frac{4}{5}$ *Answer.*

c. In division the quotient is positive if the divisor and dividend have the same sign; if the divisor and dividend have opposite signs, the quotient is negative.

Example: Divide (−15.625) by (12.5).

Solution: Since the dividend and the divisor have opposite signs, the quotient is negative.

$$\frac{-15.625}{12.5}=-1.25 \qquad \textit{Answer.}$$

d. Exercises.—Find the quotient in each of the following exercises:

(1) (−14.4)÷(0.9)
(2) (−4.32)÷(−4.8) 0.9 *Answer.*
(3) (39,483.)÷(−12.3)
(4) (1,440.)÷(0.32) 4,500 *Answer.*
(5) (1.679)÷(23)
(6) (−23.04)÷(4.8) −4.8 *Answer.*
(7) $\frac{1{,}728}{-144}$
(8) $\frac{390.59}{-28.1}$ −13.9 *Answer.*
(9) $\frac{-72.9}{-0.81}$
(10) $\frac{0.6118}{87.4}$ 0.007 *Answer.*

18. Square root.—The process of extracting square root finds many uses in solving problems. Following are the rules:

a. Begin at the decimal point and point off as many periods of two digits each as possible. Point off to the left if the number is an integer, to the right if it is a decimal. Point off to both the left and the right if there are digits both left and right of the decimal point. If the last period of the decimal has but one digit, add a cipher to complete the period.

b. Find the largest integer the square of which is equal to or less than the left-hand group, and write this integer for the first digit of the root and directly over the first group of digits.

c. Square the first digit of the root; subtract its square from the first group and bring down the second group.

d. Obtain a trial divisor by doubling the partial root already found, divide it into the remainder (omitting from the latter the right-hand digit), and write the integral part of the quotient as the next digit of the root and directly over the group of digits used in determining it.

e. Annex the root digit just found to the trial divisor to make the complete divisor; multiply the complete divisor by this root digit; subtract the result from the dividend; bring down the next group for a new dividend.

f. Obtain a new trial divisor by doubling the part of the root already found, and proceed as before until the desired number of digits of the root have been found.

g. After extracting the square root of a number involving decimals, point off one decimal place in the root for every decimal group in the number.

h. If the root is exact, square it. The result should be the original number. If the root is inexact, square it and add to this result the remainder.

Example: Find $\sqrt{681.9213}$

Solution:

	26 . 1 1
	$\sqrt{681.\ 92\ 13}$
	4
46	2 81
6	
276	2 76
521	5 92
1	
521	5 21
5221	71 13
1	
5221	52 21
	1892

(1) Point off in groups of two in both directions from the decimal point.

(2) 2 is the largest integer the square of which is equal to or less than 6, so place the 2 over the 6.

(3) Square 2 and place it under the 6. Subtract.

(4) Bring down 81, the next two digits.

(5) Bring down the 2 above the 6 and double it for the trial divisor. Divide the 4 into 28, the remainder less the last digit to the right; 7 is obtained.

```
    26.11
    26.11
 --------
    26 11
   261 1
 15666
 5222
 --------
 681.7321
     1892
 --------
 681.9213
```

(6) Annex the 7 to the trial divisor and multiply by 7; 329 is obtained. This is too large. Next annex 6 and try it. This is satisfactory. Subtract the 276 from the 281. Place the 6 above the 81 group. Bring down the next group, 92.

(7) Obtain a new trial divisor by doubling the 26. Divide the 52 into the 59; 1 is obtained. Annex the 1 and multiply. Place the 1 above the 92. Subtract and bring down the next group, 13.

(8) Obtain a new trial divisor by doubling 261. Divide the 522 in the 711; 1 is obtained. Annex the 1 and multiply. Place the 1 above 13. Subtract. 1892 is obtained as the remainder.

(9) Multiply 26.11 . 26.11 and add 1892; 681.9213 is obtained.

i. Exercises: Find the square root of the following:

(1) $\sqrt{625}$

(2) $\sqrt{289}$ 17 *Answer.*

(3) $\sqrt{125.44}$

(4) $\sqrt{167281}$ 409 *Answer.*

(5) $\sqrt{8.93}$

(6) $\sqrt{.4387}$ .66234 *Answer.*

(7) $\sqrt{983.431}$

(8) $\sqrt{10.6934}$ 3.27008 *Answer.*

19. Miscellaneous exercises.—The following exercises are based on the topics in this section:

(1) If 1 cubic foot of water weighs 62.5 pounds, what is the weight of 4.18 cubic feet of water?

(2) How many cubic feet are there in 180 pounds of water? (See *a* above.) 2.88 cu. ft *Answer.*

(3) Change the following common fractions to decimal fractions:

$$\frac{3}{8};\ \frac{7}{16};\ \frac{51}{75};\ \frac{17}{32}.$$

(4) Change the following decimal fractions to mixed numbers:

1.25; 3.875; 14.375. 1¼; 3⅞; 14⅜ *Answer.*

(5) Bolts ¾ inch in diameter and 6 inches long weigh 117 pounds per hundred bolts. What is the weight of 1,200 bolts?

(6) A certain bomber can carry a bomb load of 4,500 pounds. How many 250-pound bombs can be carried? 18 *Answer.*

(7) Which is larger 13/15 or 23/28?

(8) Divide 6¾ by 2⅝. 2 4/7 *Answer.*

(9) At an altitude of 5,000 feet and at 10° C., the calibrated air speed is 190 mph. The true air speed is 206 mph. What is the percent of increase in the two readings?

(10) At an altitude of 11,000 feet and at 20° C., the calibrated air speed is 210 mph. The true air speed is 242 mph. What is the percent of increase in the two readings? 15.2 percent *Answer.*

(11) The top air speed of an aircraft at 10,000 feet is 325 mph. At 15,000 feet it is 335 mph. What is the percent of increase in the air speed?

(12) If 469 cadets are sent to primary schools in Georgia and this group represents 14 percent of the class, find the number of cadets in the class. 3,350 cadets *Answer.*

(13) If 28 cadets out of a squadron of 196 are on guard duty, what percent of the squadron is on guard duty?

(14) On a certain flight a bomber used 40.5 gallons of gasoline per hour. The time of the flight was 3 hours 48 minutes. Find the amount of gasoline used. 153.9 gal. *Answer.*

(15) An aircraft flies a distance of 160 nautical miles. Find the distance in statute miles.

(16) The temperature reading on a centigrade thermometer was 3° C. The reading increased 2° the first hour and decreased 7° the second hour. What was the final temperature reading?

−2° C. *Answer.*

(17) On a certain day, 10 temperature readings were taken on a centigrade thermometer. They were 6°, −3°, −7°, −15°, −4°, 0°, 2°, 3°, 5°, 3°. Find the average temperature reading. *Hint:* Find the sum and divide by the number of readings.

(18) Find the product in each of the following:

(−6). (−1½). (1⅓) 12 *Answer.*

$(-2)^2$. (8⅓). (−¾) −25 *Answer.*

(19) The following numbers represent the diameters of the bores on different guns: 37 mm, 3 inches, 1 inch, 155 mm, 6 inches, 75 mm. Arrange them according to size beginning with the largest one.

(20) Express a speed of 118 kilometers per hour in terms of miles per hour. 73¾ mph *Answer.*

(21) Calculate the number of square centimeters in 1 square foot.

(22) A photographic film is designed for a picture 6 by 9 centimeters; express this in inches to the nearest quarter inch.

$2\frac{1}{4}$ by $3\frac{1}{2}$ in. *Answer.*

(23) If 570 cadets are sent to primary schools in Florida, and this group represents 30 percent of the class, find the number of cadets in the class.

(24) If 24 cadets out of a squadron of 180 passed the high altitude test, what percent of the squadron passed the test?

13.3 percent *Answer.*

(25) At a certain airdrome there are 88 aircraft, consisting of bombers and interceptor aircraft. The ratio of bombers to interceptors is 3 to 8. Find the number of each kind of aircraft.

(26) What is the diameter in inches of the bore of a 75-mm gun? (This means the bore is 75 mm in diameter.) 2.95 in. *Answer.*

(27) The following numbers represent the ranges of different aircraft: 250 nautical miles; 262 statute miles; 480 kilometers; 298 statute miles; 275 nautical miles. Arrange these distances in order of magnitude starting with the largest one.

(28) A detail of 33 cadets represents 15 percent of the squadron. How many cadets are there in the squadron? 220 *Answer.*

(29) Find the difference in temperature readings of $+47°$ C. and $-5°$ C.

(30) On a certain day the lowest temperature reading was $-14°$ F. and the highest temperature reading was $+19°$ F. Find the increase in readings. 33° *Answer.*

(31) Find the values of the following:

$$(-3)^3$$
$$(-2)^2\ (-1)^3$$
$$3(-2)^3\ (-1)^2$$
$$2(\tfrac{1}{2})^2\ (4)^3$$
$$8(-1)^2\ (\tfrac{1}{2})^3$$

(32) A panel is made up of 5 plies which are $\frac{1}{4}$ inch, $\frac{3}{8}$ inch, $\frac{1}{3}$ inch, $\frac{1}{5}$ inch, and $\frac{2}{15}$ inch thick, respectively. How thick is the panel?

$1\frac{7}{24}$ in. *Answer.*

(33) Divide 1.5625 by 0.125.

(34) Multiply $2\frac{1}{4}\ \frac{8}{36}\ 2\frac{2}{3}\ 1\frac{1}{8}$. $1\frac{1}{2}$ *Answer.*

(35) Find the sum of $1\frac{1}{2}+2\frac{2}{3}-\frac{1}{4}+\frac{5}{18}$.

(36) How many strips each $\frac{3}{32}$ inch thick are in a laminated piece $1\frac{7}{8}$ inches thick? 20 *Answer.*

(37) In a squadron of 200 cadets there are 14 cadets sick. What percent of the squadron is sick?

(38) The chord of an airplane wing is 72 inches. If the center of pressure is at a point 28 percent of the distance along the chord from the leading edge, how many inches is it from the leading edge?

20.16 inches. *Answer.*

(39) The $\frac{L}{D}$ ratio for an airfoil section is $\frac{L_c}{D_c}$. Find the $\frac{L}{D}$ ratio when $L_c=0.0018$ and $D_c=0.00008$.

(40) Find the ratio of the areas of two circles having radii of 3 inches and 4 inches. (The areas are to each other as the squares of their radii.)

9/16 *Answer.*

SECTION III

ALGEBRA

20. Purpose and scope.—Algebra is the basis for all work in formulas and trigonometry. In the solution of many problems of these types the first thing that is done is to write facts in the form of an equation. To be able to handle such work, equations must be understood and the necessary background of algebraic manipulation must be thoroughly mastered. The following paragraphs contain exercises with accompanying explanations designed to give the student a working knowledge of necessary fundamentals of algebra.

21. Algebraic symbols.—*a.* In working with general formulas, it is convenient to let numbers be represented by letters. In the actual evaluation of the formulas, the specific values are substituted for the letters.

Example: $d=\frac{1}{2}gt^2$ if $g=32$, $t=5$

Solution: then $d=\frac{1}{2}\times 32\times 25=16\times 25=400$ *Answer.*

The above formula is solved for d. If it became necessary to solve for some other letter in the formula, other operations would be required. To perform such operations, it is essential to learn some of the fundamental characteristics of algebra.

b. In algebra, because of the use of x as a letter, new signs are adopted to indicate multiplication.

(1) $4x$ means 4 times x. When no sign appears between letters and numbers, multiplication is indicated.

(2) $4 \cdot x$ means 4 times x. When some sign of multiplication is necessary, a dot is used as indicated.

(3) In any product such as the foregoing, the coefficient of the x term is defined to be the rest of that term.

(4) When there is a product of the type $x \cdot x \cdot x \cdot x$, it is written x^4. The 4 is called the exponent of x, and x^4 is called a power of x. This is simply a shorthand method of writing a product. The x is termed the base.

Example: What are the coefficients and exponents of—

$4x^2$? x^2? $4x$? x?

Solution:

4 is the coefficient and 2 is the exponent.
1 is the understood coefficient and 2 is the exponent.
4 is the coefficient and 1 the understood exponent.
1 is understood as the coefficient and exponent.

c. *Rules for use of exponents.*

(1) When two powers with the same base are multiplied together, the exponents are added, as shown in the following illustrations:

$$3^2 \cdot 3^3 = 3^5 = 243$$
$$(9 \cdot 27 = 243)$$
$$x^2 \cdot x^4 = x^6$$

(2) When two powers with the same base are divided, the exponents are subtracted, as the following examples show:

$$\frac{2^5}{2^2} = 2^3 = 8$$
$$\left(\frac{32}{4} = 8\right)$$
$$\frac{x^7}{x^3} = x^4$$

(3) When two products are multiplied together, the coefficients are multiplied and each power is multiplied separately, as in the following examples:

(*a*) $2x \cdot 3x^2 = 6x^3$.

(*b*) $2abx \cdot 5a^2x = 10a^3bx^2$.

d. Terms are products or quotients separated from each other by + and − signs. For example, in the expression $6x^2y-2ax+4ab$, the terms are $6x^2y$, $-2ax$, and $4ab$.

e. A polynomial is a sum of two or more terms which do not involve division. To illustrate:

$$4x^3+x^2-1 \text{ is a polynomial}$$

$$a+b+c-1 \text{ is a polynomial}$$

$$\frac{x^5-x^3+x}{x-1} \text{ is not a polynomial}$$

$$x^4-2x^3+\frac{1}{x}-7 \text{ is not a polynomial}$$

22. Addition and subtraction of polynomials.—*a*. To add or subtract polynomials, the terms should be arranged so that like terms fall in columns. Then these terms should be added or subtracted according to the rules of signed (positive and negative) numbers. Only those terms that are exactly alike in all respects except for the numerical coefficient, may be added or subtracted. For example, x and x^2 cannot be combined into a single term.

(1) *Example:* Add x^3-3x^2+1, x^3+x-3, and x^2+x+1.

Solution:

$$\begin{array}{rrrr} x^3 & -3x^2 & & +1 \\ x^3 & & +x & -3 \\ & x^2 & +x & +1 \\ \hline 2x^3 & -2x^2 & +2x & -1 \end{array} \qquad \textit{Answer.}$$

NOTE.—It is advisable, if possible, to arrange polynomials in order of decreasing exponents of the unknown.

(2) *Example:* Subtract x^3+3x^2+x-1 from x^4+x^3-x+2

Solution:

$$\begin{array}{rrrrr} x^4 & +x^3 & & -x & +2 \\ & x^3 & +3x^2 & +x & -1 \\ \hline x^4 & & -3x^2 & -2x & +3 \end{array} \qquad \textit{Answer.}$$

b. *Exercises.*—Add the following polynomials:

(1) $5x+5y$, $6x-8y+5z$, $5x-3y-3z$.

(2) x^2-x-2, $2x-7x^2+4$, $6x^2+x-7$. $2x-5$ *Answer.*

(3) $2x+3$, $4x-7$, $x-2$.

(4) $2x+2y+\frac{1}{2}z$, $2\frac{1}{2}x-3\frac{1}{2}y+2z$, $x-y-z$.

$5\frac{1}{2}x-2\frac{1}{2}y+1\frac{1}{2}z$ *Answer.*

(5) $12bc+ac$, $9\,ab-bc$, $14bc-3ab$.

(6) $2a+2c+x-5$, $2a+3x+8-3c$, $a-9-3c-x$, $16-13x-14a-c$.

$-9a-5c-10x+10$ *Answer.*

c. *Exercises.*—Subtract the following polynomials:

(1) $2a-9$ from $7a-11$.

(2) x^2-3x-5 from $2x^2-7x+9$. $x^2-4x+14$ *Answer.*

(3) $3x^3-5$ from $-7x^3+4x^2$.

(4) x^2+2x-3 from $5x^2-4x-7$. $4x^2-6x-4$ *Answer.*

(5) $4a^2x-3a^2y+xy$ from 0.

(6) $2x^5-4x^3-5$ from $x^4+2x^2-3x^3-x^5$.

$-3x^5+x^4+x^3+2x^2+5$ *Answer.*

23. Multiplication and division of polynomials.—*a*. To multiply two polynomials, multiply one of them by each term of the other separately, then combine the results, following the rules of addition.

Example: x^3-2x^2-x-1 by x^2+2.

Solution:

$$\begin{array}{r} x^3+2x^2-x-1 \\ x^2+2 \\ \hline x^5+2x^4-x^3-x^2 \qquad\qquad \\ 2x^3+4x^2-2x-2 \\ \hline x^5+2x^4+x^3+3x^2-2x-2 \end{array}$$

Answer.

If three or more polynomials are to be multiplied together, multiply two of them and then multiply this product by the third, etc.

b. To divide one polynomial by another.—(1) Arrange the dividend and the divisor according to descending powers of one variable, starting with the highest powers at the left.

(2) The result of dividing the first term of the dividend by the first term of the divisor is the first term of the quotient. Ignore the rest of the terms for the time being.

(3) Multiply the entire divisor by this first term of the quotient and subtract the result from the dividend.

(4) The second term of the quotient is obtained by dividing the first term of the remainder by the first term of the divisor.

(5) Using this second term, multiply the divisor and subtract from the remainder as before.

(6) Continue this process until the first term of the divisor cannot go into the first term of the last remainder. This remainder is

written after the quotient as the numerator of a fraction, of which the divisor is the denominator.

(7) To check division, multiply the divisor and quotient. This should be equal to the dividend.

Example: Divide $-5x^2-13x+3x^3+10$ by $3x-2$.

Solution: Rearrange in order of descending powers, before dividing:

$$3x^3-5x^2-13x+10 \text{ by } 3x-2$$

$$\begin{array}{r|l|l} 3x-2 & 3x^3-5x^2-13x+10 & x^2-x-5 \\ & \underline{3x^3-2x^2} & \\ & \quad -3x^2-13x & \\ & \quad \underline{-3x^2+2x} & \\ & \qquad -15x+10 & \\ & \qquad \underline{-15x+10} & \end{array}$$

Answer.

c. *Exercises.*—Multiply the following polynomials:

(1) $(a+2)\ (a+5)$.

(2) $(x+y)\ (x-y)$. x^2-y^2 *Answer.*

(3) $(3x^2-12x+18)\ (x-4)$.

(4) $(x^2-4x-12)\ (x^2+4x+2)$. $x^4-26x^2-56x-24$ *Answer.*

(5) $(x^2-7x-12)\ (\mathrm{b}x-2b)$.

(6) $(x4-9y^2)\ (x^3-3y)$. $x^7-3x^4y-9x^3y^2+27y^3$ *Answer.*

d. *Exercises.*—Divide the following polynomials:

(1) $(a^2+5a+6)\div(a+2)$.

(2) $(x^2-10x-39)\div(x+3)$. $x-13$ *Answer.*

(3) $(2x^2-13x+20)\div(2x-5)$.

(4) $(x^4-x^3-4x^2-x+1)\div(x^2+2x+1)$. x^2-3x+1 *Answer.*

(5) $(x^6-y^6)\div(x^2-y^2)$.

(6) $(x^7+1)\div(x+1)$. $x^6-x^5+x^4-x^3+x^2-x+1$ *Answer.*

24. Evaluation of algebraic expressions.—*a.* To evaluate algebraic expressions, substitute the numerical values for the literal values and perform all indicated operations. In any series of operations, multiply and divide before adding and subtracting. Clear parentheses by doing work inside the parentheses first.

(1) *Example:* Evaluate $ax^2+2bx-4bc$ when $a=1$, $b=3$, $x=-2$, $c=4$.

Solution: $[1\cdot(-2)^2]+[2\cdot3\cdot(-2)]-(4\cdot3\cdot4)=$

$[1\cdot4]+[6\cdot(-2)]-[12\cdot4]=$

$4-12-48=-56$ *Answer.*

(2) *Example:* Evaluate $a[(a-b)+c(a^2-c)-d(a-c^2)]$ when $a=2$, $b=3$, $c=4$, $d=6$.

Solution: $2[(2-3)+4(2^2-4)-6(2-4^2)]=$

$2[-1+4(0)-6(-14)]=$

$2[-1+0+84]=2[83]=166$ *Answer.*

b. Exercises.—Evaluate the following if $x=3$, $y=2$, $z=5$.

(1) $x(y+2)(3y+x)$

(2) $4(x^2-y)+y^2(z-y)$ 40 *Answer.*

(3) $(y-3x)+z(y-x)$

(4) $x(x+y)+z^2+x(y-3)$ 37 *Answer.*

(5) $0[x+y(z-7)+4]-7$

(6) $\frac{z-x}{z-y}$ $\frac{2}{3}$ *Answer.*

(7) $\frac{x+y}{y}$

25. Equations.—*a.* An equation is a statement of equality between any two quantities. Thus, in reality, all formulas are equations. In most equations that are used, there is an unknown quantity for which a value is sought. To find this, certain rules must be followed:

(1) Equal quantities may be added to both sides of the equation.

Example:
$$\begin{array}{r} x-4=\ \ 7 \\ +4=+4 \\ \hline x\quad =\ 11 \end{array}$$

(2) Equal quantities may be subtracted from both sides of the equation.

Example:
$$\begin{array}{r} x+2=\ \ 5 \\ -2=-2 \\ \hline x\quad =\ \ 3 \end{array}$$

(3) Both sides of the equation may be multiplied by the same quantity.

Example:
$$\frac{x}{2}=4$$
$$x=4.2$$
$$x=8$$

(4) Both sides may be divided by the same quantity (zero excepted).

Example:
$$3x=12$$
$$\frac{3x}{3}=\frac{12}{3}$$
$$x=4$$

(5) The above solutions may readily be checked and proved correct by substituting in the original equation the value found for x.

b. It is possible to shorten these operations by shifting any term from one side of the equal sign to the other and changing its sign. This is called *transposition.*

c. An advanced type of equation that it is necessary to know how to solve is the *fractional* equation. In this type one added operation must be performed before applying the preceding rules. The first step in such equations is to find the least common denominator and with this quantity multiply both members through term by term.

Example: Solve the following equation:

$$\frac{x}{2}-\frac{5x}{6}=2-\frac{1}{9}(x-2)$$

Solution: $\frac{x}{2}-\frac{5x}{6}=2-\frac{(x-2)}{9}$ (L. C. D.$=18$)

$$18\cdot\frac{x}{2}-18\cdot\frac{5x}{6}=18\cdot 2-\frac{18(x-2)}{9}$$

$$9x=15x=36-2x+4$$

$$9x-15x+2x=36+4$$

$$-4x=40$$

$x=-10$ *Answer.*

d. Exercises.

(1) $5x-3=3x+3$

(2) $3x+5+x+3=0$ $x=-2$ *Answer.*

(3) $6x+4=x-16$

(4) $5n-4+6n-40=0$ $n=4$ *Answer.*

(5) $5x=\frac{3}{4}$

(6) $\frac{x}{3}-4=5$ $x=27$ *Answer.*

(7) $\frac{x-1}{2}=3+x$

(8) $\frac{4}{2x+2}=\frac{5}{3x+2}$ $x=1$ *Answer.*

(9) $\frac{x}{2}+\frac{x}{3}=10$

(10) $\frac{5x-3}{8}-\frac{3x-6}{5}=1$ $x=7$ *Answer.*

(11) $\frac{1}{x+3}+\frac{1}{x+6}=\frac{2}{x+4}$

(12) $\frac{2}{x-2}+\frac{2}{x+4}=\frac{4}{x-3}$ $x=\frac{5}{4}$ *Answer.*

e. Formulas and exercises.

(1) The horsepower required for flying an airplane is found by the formula:

$$HP=\frac{DV}{375},$$

where HP=horsepower required
D=total drag of the airplane in pounds
V=velocity in miles per hour and 375 is a constant

Find the horsepower required if $D=250$ pounds and $V=285$ mph.

(2) The general gas law is—

$$\frac{PV}{T}=\frac{P_1V_1}{T_1}$$

where P=initial pressure
V=initial volume
T=initial absolute temperature (273°+° C.)
P_1=new pressure
V_1=new volume
and T_1=new absolute temperature

Solve the above equation for V_1.

Solution:

$$\frac{PV}{T}=\frac{P_1V_1}{T_1}$$

Multiply each side of the equation by TT_1:

$$PVT_1=P_1V_1T$$

or

$$P_1V_1T=PVT_1$$

Divide each side of the equation by P_1T:

$$V_1=\frac{PVT_1}{P_1T}$$ *Answer.*

(3) In exercise (2) if $P=15$ pounds/square inch, $T=7°$ C., $P_1=20$ pounds/square inch, $V_1=450$ cubic inches, and $T_1=27°$ C., find the value of V.

(4) The formula $C=\frac{5}{9}(F-32)$ is used in the conversion of temperature readings from the Fahrenheit scale to the centigrade scale. If

the temperature reading is 86° on the Fahrenheit scale, what would the temperature reading be on the centigrade scale?

Solution:

$$C=\frac{5}{9}(F-32)$$

$$C=\frac{5}{9}(86-32)$$

$$C=\frac{5}{9}(54)$$

$$C=30^\circ$$ *Answer.*

(5) (*a*) Using the formula in (4) above, find the Fahrenheit reading when the centigrade reading is 20°.

(*b*) When will the Fahrenheit and centigrade readings be equal? (Negative values may be used.)

(6) The formula for determining the best propeller diameter for maximum efficiency is—

$$D=\frac{V}{1.03n}$$

where D=propeller diameter in feet
V=velocity of airplane in ft/sec.
n=revolutions/sec.
and 1.03 is a constant

Determine the propeller diameter when V=210 mph and n=30 revolutions/second.

Solution: $$210 \text{ mph}=\frac{(5280)(210)}{3600} \text{ ft/sec.}=308 \text{ ft/sec.}$$

$$D=\frac{308 \text{ ft/sec.}}{1.03(30 \text{ revolutions/sec.})}=9.97 \text{ ft}$$ *Answer.*

(7) Using the formula in exercise (6), determine the propeller diameter when V=150 mph and n=20 revolutions/second.

(8) The horsepower necessary to propel an airplane is proportional to the cube of the velocity. If 120 horsepower is required to fly an airplane at 130 mph, how many horsepower would be required to fly it at 150 mph?

Solution: $$\frac{HP_1}{HP_2}=\frac{V_1^3}{V_2^3}$$

$$\frac{120}{HP_2}=\frac{(130)^3}{(150)^3}$$

$$(130)^3 \cdot HP_2=120(150)^3$$

$$HP_2=\frac{120(150)^3}{(130)^3}$$

$$HP_2=184.3 \text{ hp}$$ *Answer.*

(9) On the basis of the information in exercise (8), how many times would the horsepower have to be increased to double the velocity?

(10) An airplane flying 160 mph covers a certain distance in 2 hours 30 minutes. How long would it take it to cover the same distance when flying 200 mph? 2 hr *Answer.*

26. Word problems.—*a*. One particular type of word problem is of special importance to an aviation cadet—the type involving time, rate, and distance.

(1) The following formula always holds true in such problems:

$$d=rt$$

where d=distance, r=rate, and t=time.

(2) By applying algebraic processes, this formula may also be written in the following forms:

$$r=\frac{d}{t} \qquad t=\frac{d}{r}$$

(3) Most problems of this type may be solved by employing one of the three forms of this equation.

b. In solving word problems, be sure to be specific about what the unknowns are to represent. Try to develop the equation so that a distance, time, or rate is expressed in terms both of known quantities and unknown quantities. These may then be set equal to each other. The following rules give a procedure that can be used in solving most word problems.

Rule 1. Pick one of the unknown quantities to be determined and represent it by a letter.

Rule 2. Express the unknown quantities in terms of the chosen letter, using the relationships given in the problem.

Rule 3. Find which quantities are implied to be equal by the statement of the problem, and set them equal to obtain an equation.

Rule 4. Check the exact kind of units used in expressing all quantities and be sure only like units are equated.

(1) *Example:* A pursuit plane flies 90 mph faster than a bomber in still air. The bomber travels 42 miles while the pursuit plane travels 56 miles. Find the average speed of each aircraft.

Solution: The unknowns to be determined are the speeds. Picking one of these (Rule 1), let—

r equal average speed of the bomber in mph. Then, in terms of r, the speed of the pursuit ship can be expressed (Rule 2) as $r+90$ equals the average speed of the pursuit ship in mph.

From the fact that time is equal to distance divided by rate, the unknown time each ship took can be expressed thus (Rule 2):

$42/r$ = time in hours for bomber.

$56/r+90$ = time in hours for pursuit ship.

The problem states that the two times are equal, hence (Rule 3) $42/r=56/r+90$.

This equates hours to hours (Rule 4).

Cross-multiplying the equation gives

$$56r=42r+3{,}780$$

or

$$56r-42r=3{,}780$$

hence

$$14r=3{,}780$$

and $r=270$ mph, the speed of the bomber *Answers.*

consequently $r+90=360$ mph, the speed of the pursuit ship.

(2) *Example:* Two airplanes start out toward each other from two towns 570 miles apart. They meet after 2 hours. If the speed of one is 15 mph more than the speed of the other, find the speed of each.

Solution: Analysis of the problem reveals that the total distance traveled by the two airplanes is 570 miles. This distance may be set up in terms of the unknown rates and known times.

Let x = mph of slower airplane. Then $x+15$ = mph of other airplane. Since $d=rt$, set up the distances in terms of the unknown and place them equal to the known distance:

$$2x+2(x+15)=570$$

$$2x+2x+30=570$$

$$4x=570-30$$

$$4x=540$$

$$x=135 \text{ mph}$$ *Answers.*

$$x+15=150 \text{ mph}$$

(3) *Example:* An airplane flew from March Field to Moffett Field at the rate of 150 mph. It returned the following day at the rate of 200 mph and required 40 minutes less time to make the trip. How far is it from March Field to Moffett Field?

Solution: The fact used in making the equation in this case is that the distance is the same both ways. The distance will not be solved for directly.

Let x = number of hours for trip from March Field to Moffett Field.

Then $x-\frac{2}{3}$=number of hours for return trip. (Note time is changed to hours.)

$$150x=200(x-\frac{2}{3})$$

$$150x=200x-\frac{400}{3}$$

$$-50x=-\frac{400}{3}$$

$$x=\frac{8}{3}$$

$$\text{Distance}=\frac{8}{3}\cdot 150=400 \text{ miles}$$ *Answer.*

c. One other type of word problem in algebra has considerable importance in aviation. An example will be worked to show the type:

Example: Three observation airplanes, 0–1, 0–29, and 0–5, were able to map a certain region in 3 hours. A week later, plane 0–29 mapped the region alone in 8 hours. Two weeks later plane 0–1 was sent out and it took 10 hours to do the job. Assuming all conditions equal, how long should it take plane 0–5 to map the entire region alone?

Solution: This type of problem may be solved by finding the fractional part of the job completed by each separate airplane in a certain time and setting the sum of these fractions equal to the fraction of the total job completed in this time.

Let x = number of hours necessary for 0–5 to do the job alone. In 1 hour each airplane does the following:

$$\frac{1}{8}, \frac{1}{10}, \frac{1}{x}$$

Add these and set equal to the part of the job completed together.

$$\frac{1}{8}+\frac{1}{10}+\frac{1}{x}=\frac{1}{3}$$

This fractional equation may be solved by using the common denominator of $120x$:

$$15x+12x+120=40x$$

$$120=13x$$

$$x=9\frac{3}{13}$$ *Answer.*

d. Exercises.

(1) A certain bomber can fly 170 mph loaded and 190 mph empty. If it leaves the base at 8 PM and must be back by 5 AM, how far from

the base can it go if it unloads all its bombs and comes right back empty?

(2) Two airplanes start out toward each other from two towns 680 miles apart. They meet after 2 hours. If the speed of one is 20 mph more than the speed of the other, find the speed of each.

160 and 180 mph *Answer.*

(3) Three observation airplanes, 0–1, 0–2, and 0–3, were able to map a certain region in 3 hours. Plane 0–1 can do the job alone in 9 hours, and plane 0–2 can do the job alone in 6 hours. Assuming all conditions equal, how long should it take plane 0–3 to map the region alone?

(4) An airplane flew from town A to town B at a rate of 180 mph. It returned at a rate of 210 mph and required 20 minutes less time for the return trip. Find the distance from A to B.

420 miles *Answer.*

(5) An airplane leaves its base and flies due east at a rate of 175 mph. Fifteen minutes later another airplane takes off in pursuit of the first airplane at a rate of 210 mph. How long does it take for the second airplane to catch the first airplane and how far from the base does this take place?

(6) If planes 0–1, 0–3, and 0–5 can each map a region alone in 10, 12, 15 hours, respectively, find how long it would take them to do the job together. 4 hr *Answer.*

27. Miscellaneous exercises.

(1) Solve for x:

$$4x-3=5x+9$$

(2) Evaluate when $a=2$, $b=3$, $c=5$:

$$ab\ [a(a-c)-a^3b^2(b-c)+a^2b-b^3c]$$

90 *Answer.*

(3) Three observation airplanes, 0–4, 0–7, and 0–9, wished to map a region. Each could do the work alone in 4, 6, and 8 hours, respectively. How long will it take all three airplanes to do the job together?

(4) Solve for x:

$$\frac{1}{x}+2=\frac{8}{x}-\frac{1}{3}$$ $x=3$ *Answer.*

(5) Solve for x:

$$\frac{3}{x+1}-\frac{3}{x+5}=0.$$

(6) One airplane can fly from San Diego to Victorville in 1½ hours. Another airplane can make the trip in 2 hours. If they start from opposite ends of the course at the same time and fly toward each other, after what time will they meet? 6/7 hr *Answer.*

(7) An airplane flying a course against the wind covers 160 miles in 1 hour. At a later time the airplane flies the return trip with the wind, with a ground speed of 200 miles per hour. If the round trip took 5 hours, what was the round-trip distance?

(8) Solve for x:

$$\frac{3x}{2}-\frac{4}{3}(x-5)-\frac{1}{6}(5x-12)=0 \qquad x=13 \quad \textit{Answer.}$$

(9) Evaluate when $x=-1$, $y=4$, $z=2$:

$$xy\cdot x^2+4(yz+x)-x^3+yz^2$$

(10) One observation airplane can map a certain region in 1 hour. A second airplane can map the region in 1½ hours. How long will it take the two to do it together. ⅗ hr *Answer.*

(11) A ferry pilot travels 800 miles by airplane and 100 miles by train in 4 hours 40 minutes. Then, at the same speeds as before, he flies 700 miles in another airplane and 150 miles by train in a total of 5 hours 20 minutes. What were the speeds of the airplanes and trains?

Section IV

SCALES

28. Scope.—The word "scale" is used in this section as in "*scale* model," the "*scale* of a map," the "*scale* of a drawing," and so on. The practical use of scales in connection with maps, drawings, and silhouettes is illustrated by examples and exercises.

29. Models.—*a.* A true scale model of an airplane, for example, is a model which has been constructed so that the ratio of the length of *any* part of the model to the actual length of the same part of the airplane is the same for all parts. Thus, if the wing span on the model is 5 inches, and the wing span of the actual airplane is 55 feet, then 1 inch anywhere on the model represents 11 feet or 132 inches. Then this is the *scale* of the model: 1 to 132, or $\frac{1}{132}$. When a scale is stated simply as 1 to 132 it means that 1 unit of any kind on the model represents 132 of the same kind of units on the airplane. In other words, the model is $\frac{1}{132}$ the size of the airplane.

b. Example: The U. S. Government is encouraging youths to build scale models of various aircraft. The *scale* to be used is the same for all aircraft: 1 to 72. The wing span of the German Heinkel bomber

(He–111K Mk111) is 76 feet. What will be the wing span of the model?

Solution: 76 feet=76×12 inches

$$\frac{\text{wing span (model)}}{76\times 12\text{ in.}}=\frac{1}{72}$$

Therefore, wing span of model$=\frac{76\times 12}{72}=\frac{76}{6}=12\tfrac{2}{3}$ in. *Answer.*

c. Exercises.—The following models are all constructed to the scale of 1 to 72:

(1) The model wing span of a B–18 is 15 inches. What is the actual wingspan?

(2) The over-all length of a Messerschmitt (Me–110) is 36 feet. How long will the model be? 6 in. *Answer.*

(3) The over-all length of a model of a B–23 is 8⅝ inches. What is the over-all length of the B–23 airplane?

30. Maps.—*a.* A map is a scale diagram to show the disposition of geographic features on the earth such as cities, roads, rivers, etc. On most maps, the scale used is conveniently stated by a diagram as in figure 23. It may also be expressed as a ratio, for example: 1 to 500,000.

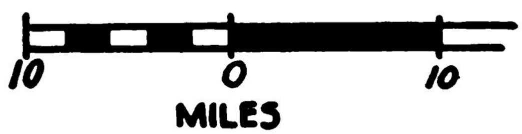


FIGURE 23.

b. Of primary interest to airmen are aeronautical charts, or maps. The sectional charts of the United States are made to a scale of 1 to 500,000. The regional charts of the United States are made to a scale of 1 to 1,000,000.

Example: What distance (miles) does 1 inch represent on a sectional chart?

Solution: Since the scale is 1 to 500,000, then 1 inch represents 500,000 inches on the earth. 1 mile = 5,280 feet = 5,280 × 12 inches. Therefore

$$500{,}000\text{ inches} = \frac{500{,}000}{5{,}280\times 12}\text{ miles} = 7.9\text{ miles.}$$

1 inch represents 7.9 miles *Answer.*

31. Miscellaneous exercises.

(1) On a regional chart 1 inch = how many miles?

(2) The aeronautical planning chart of the United States (3060a) is drawn to a scale of 1 to 5,000,000. On this chart, a distance of 2¼ inches is the same as how many miles? 177.5 miles. *Answer.*

(3) By direct measurement, determine the scale for the map in figure 24. From Catskill to Albany is 30 miles.

(4) How far is Schenectady from Albany? (See fig. 24.) Use the scale determined in exercise (3) above. 16 miles. *Answer.*

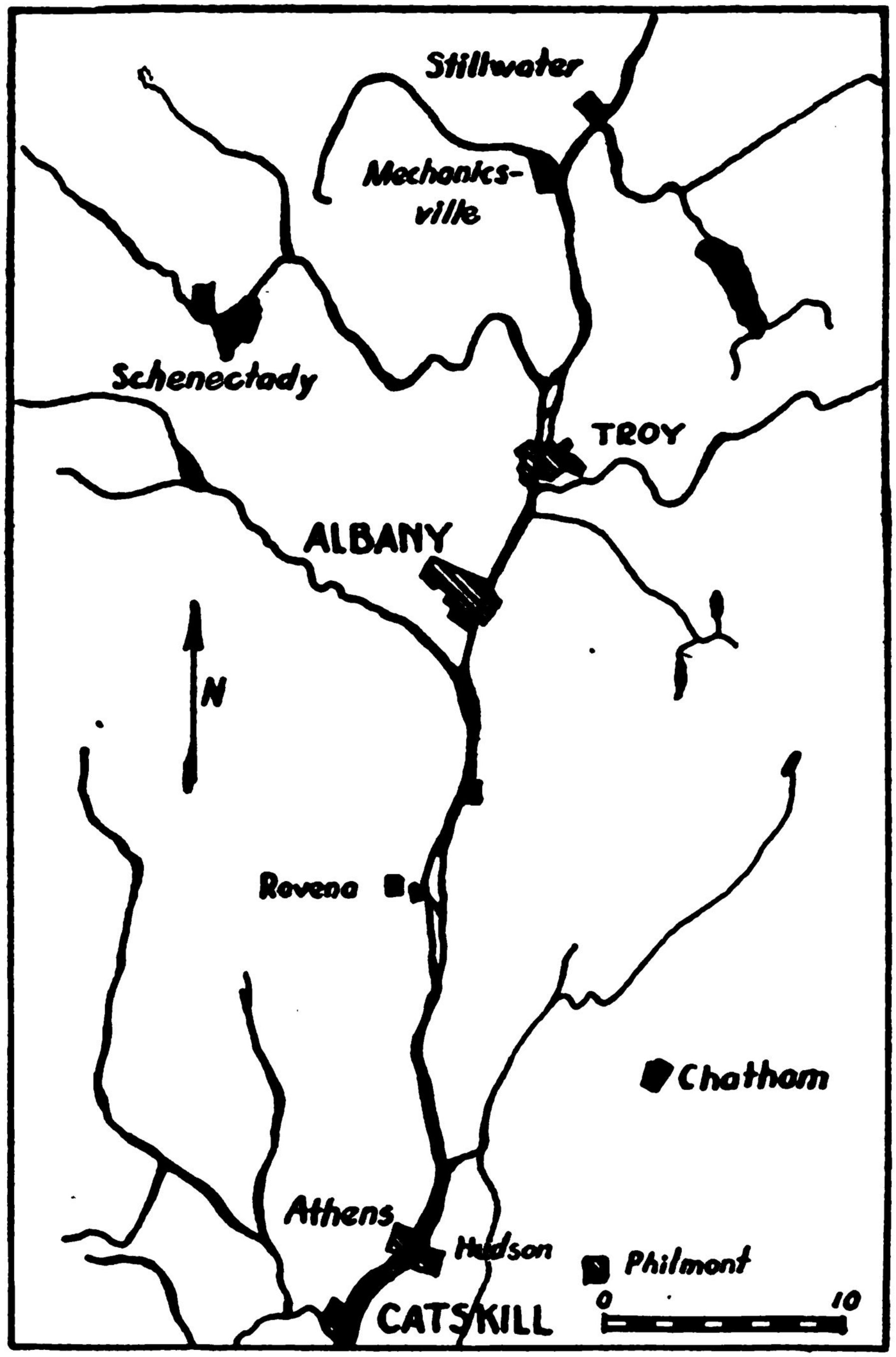


FIGURE 24.

(5) By direct measurement, determine the scale used for the map in figure 25. See exercise (3) above.

(6) Is Chatham located properly in figure 25? Why? (Its location is correct in figure 24.) No. *Answer.*

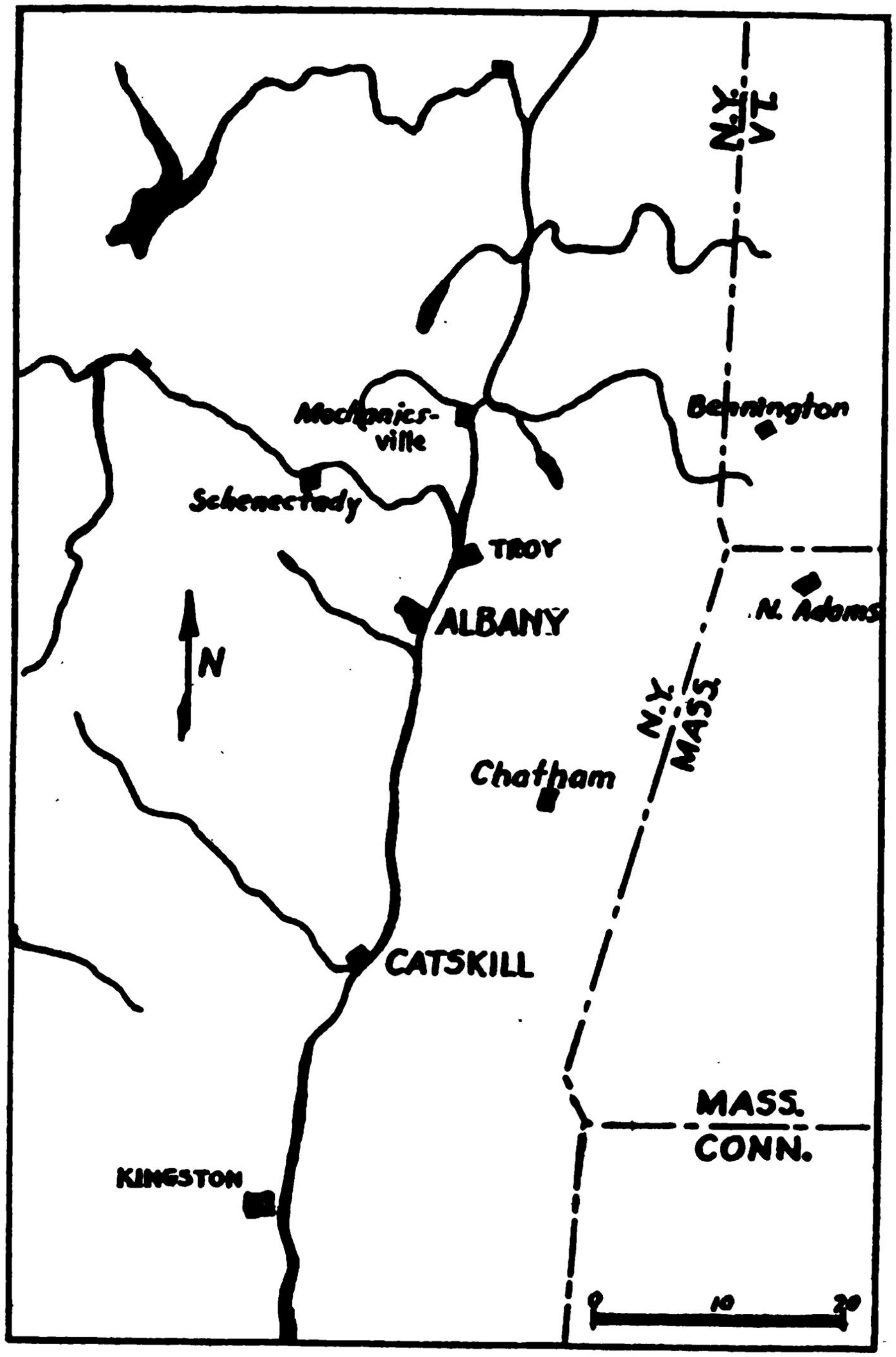


FIGURE 25.

(7) The model of the German bomber Heinkel (He–177) has a wing span of 17⅙ inches. What must be its actual wing span?

(8) The airplane models described in paragraph 29*b* will be used for training gunners in range determination. How far from the model should a gunner be so that it will appear to him as though the actual airplane were 600 yards away?

Solution: His distance from the model should be in the same ratio to 600 yards as the model scale, or as 1 is to 72:

$$\frac{\text{distance}}{600\text{ yd}}=\frac{1}{72}, \text{ or distance}=\frac{600\text{ yd}}{72}=8\tfrac{1}{3}\text{ yd}=25\text{ ft}. \qquad \textit{Answer.}$$

(9) A top view silhouette of a B–23 is to be drawn as large as possible in a space 2 feet wide. The wing span of a B–23 is 91 feet. Which of the following scales should be used: 1 to 60, 1 to 100, 1 to 20, or 1 to 50?

(10) Using the information in exercise (3) above, are the scales shown in figures 24 and 25 correct?

Section V

GRAPHS

Paragraph

32. Purpose.—*a*. Graphs are used to represent pictorially the relationship between two quantities, that is, how one quantity varies with another quantity. The following example shows how the pressure of a gas varies with the volume at a temperature of 0° C:

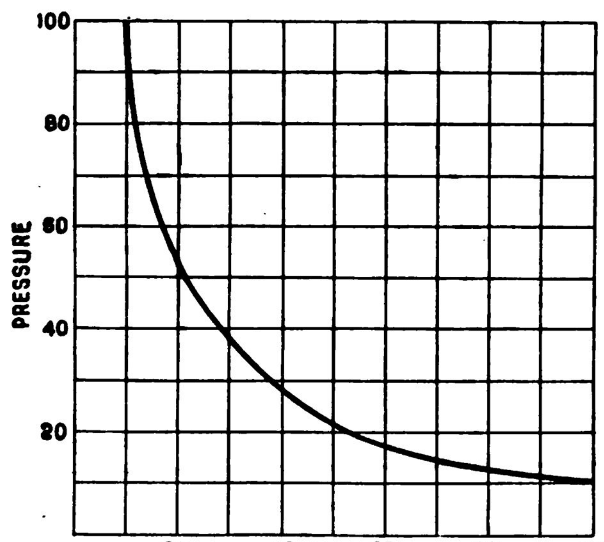


FIGURE 26.

b. Graphs are used extensively to portray certain physical relations. Once the graph has been drawn, it can be used to make rapid approximate computations, thus saving valuable time and work. For this reason, the graph is of great value in aeronautics.

c. To interpret fully the graphic process, it is necessary to understand the construction of graphs. In illustrating their construction, the procedure used by mathematicians will be presented briefly and applied to aeronautical information.

33. Axes and points.—*a.* Begin by taking two known straight lines intersecting at right angles in the same plane. The point of intersection is called the origin, *O*. The straight lines *OX* and *OY* are reference lines and are known as rectangular axes.

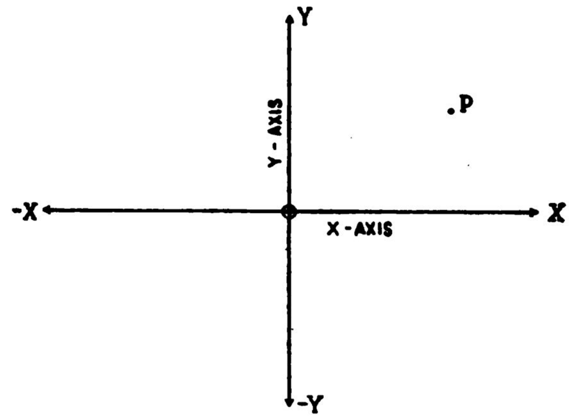


FIGURE 27.

b. The horizontal line is called the *X*-axis. The vertical line *OY* is called the *Y*-axis.

c. The position of any point is fixed by reference to these two axes. Thus, the point *P* (fig. 27) is located by describing it as so many units horizontally from *OY* (in the *X*-direction) and so many units vertically from *OX* (in the *Y*-direction). In any graph, certain units are marked off on the *X*-axis, and the same or different units are marked off on the *Y*-axis. (See figs. 28 and 29.)

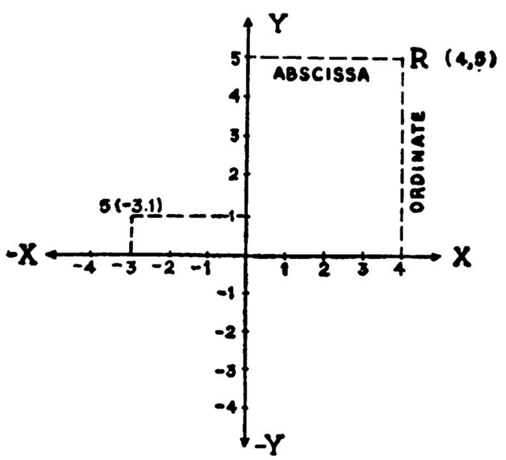


FIGURE 28.

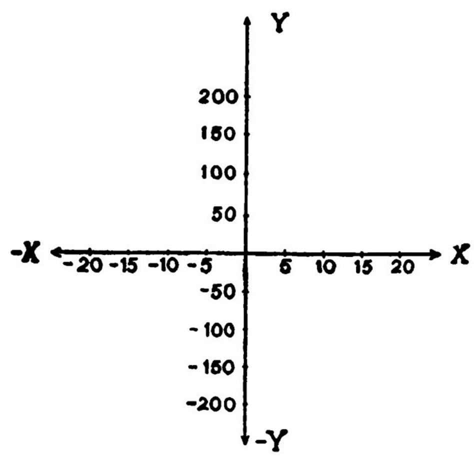


FIGURE 29.

d. It is to be noted that in going from *OY* to the right in the *X*-direction, the numbers are positive, signifying the positive direction; while to the left the numbers are negative, signifying the negative direction. On the other hand, going from *OX* upward in the *Y*-direction, the numbers are positive, signifying the positive direction; while downward the numbers are negative, signifying the negative direction. Hence it takes two numbers, the coordinates, to locate a single point: one of them the Y-value, denoting the vertical distance, and the other, the X-value, denoting the horizontal distance. The word *ordinate* is sometimes used to designate the Y-value, and the word *abscissa* is sometimes used to designate the X-value. In figure 28, the point R is located as being 4 units horizontally from OY (4 units in the X-direction or $X=4$) and 5 units vertically from OX (5 units in the Y-direction or $Y=5$). Symbolically, the point is described by its coordinates (4, 5) meaning $X=4$, and $Y=5$. The coordinates of the point S are $(-3, 1)$ meaning $X=-3$, and $Y=1$: that is, 3 units to the left of OY in the X-direction and 1 unit above OX in the Y-direction.

e. This practice of plotting the position of a point by coordinate axes is used in locating the latitude and longitude of a point in a Mercator Chart.

34. Reading graphs.—To illustrate the procedure, suppose it is desired to know the calibrated air speed corresponding to an indicated air speed of 150 mph on the meter the calibration curve of which is shown in figure 30. From "150" on the horizontal axis, move up until the curve is intersected. Then move horizontally to the left. On the vertical axis read "158." This is the corresponding calibrated air speed in miles per hour. All graphs are read by following this general procedure, although occasionally some graphs may be complicated by several curves and multiple scales.

a. Air speed meter calibration graph.—This graph (fig. 30) is based upon an air speed meter calibration of an airplane.

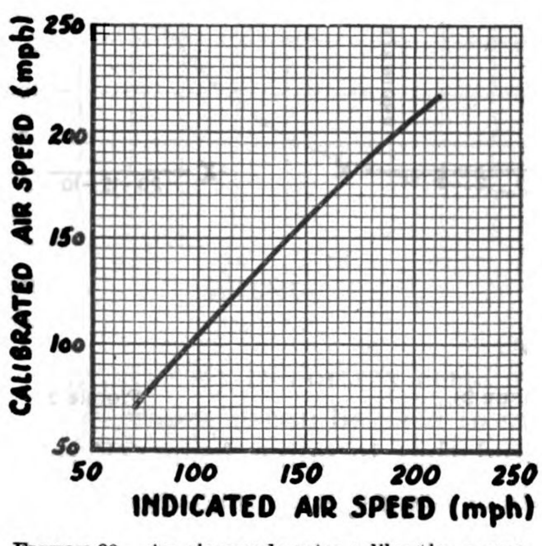


FIGURE 30.—An air speed meter calibration curve.

(1) If the indicated air speed is 110 mph, find the calibrated air speed.

(2) If the indicated air speed is 165 mph, find the calibrated air speed. 173 mph *Answer.*

(3) If the calibrated air speed is 198 mph, find the indicated air speed.

b. Pressure-temperature graph.—This graph (fig. 31) shows the relationship between the pressure and the temperature of a gas at constant volume.

(1) At a temperature of 0° C., the pressure is__________________.

(2) If the temperature increases, the pressure__________________.

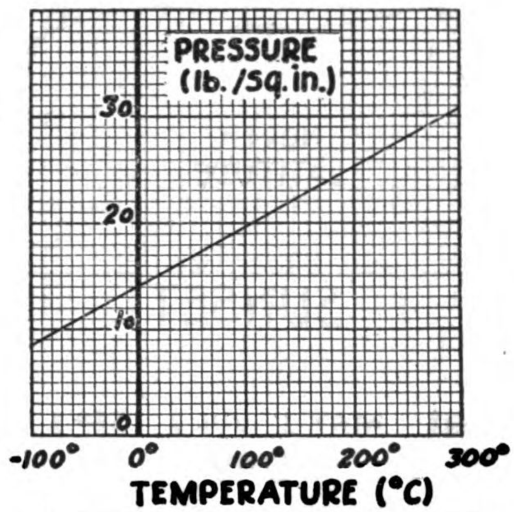


FIGURE 31.—Pressure and temperature of a gas at constant volume.

(3) If the pressure is decreased, the temperature of the gas_________.

(4) At a temperature of 100° C., the pressure is _____________.

(5) If the pressure is 30 pounds/square inch, the temperature is _____________.

c. 24-hour system.—The 24-hour system for stating time eliminates the use of the abbreviations AM and PM. The values for AM time are unchanged except that four figures are always used. For example, 9:15 AM becomes 0915 hour; 4:45 AM becomes 0445 hour, and 11:48 AM becomes 1148 hour. The values for PM time are increased by 1200, hence 1:30 PM becomes 1330 hour and 5:55 PM becomes 1755 hour. The use of this system decreases the chances for making errors and for this reason it has been adopted for use in the U. S. Army Air Forces.

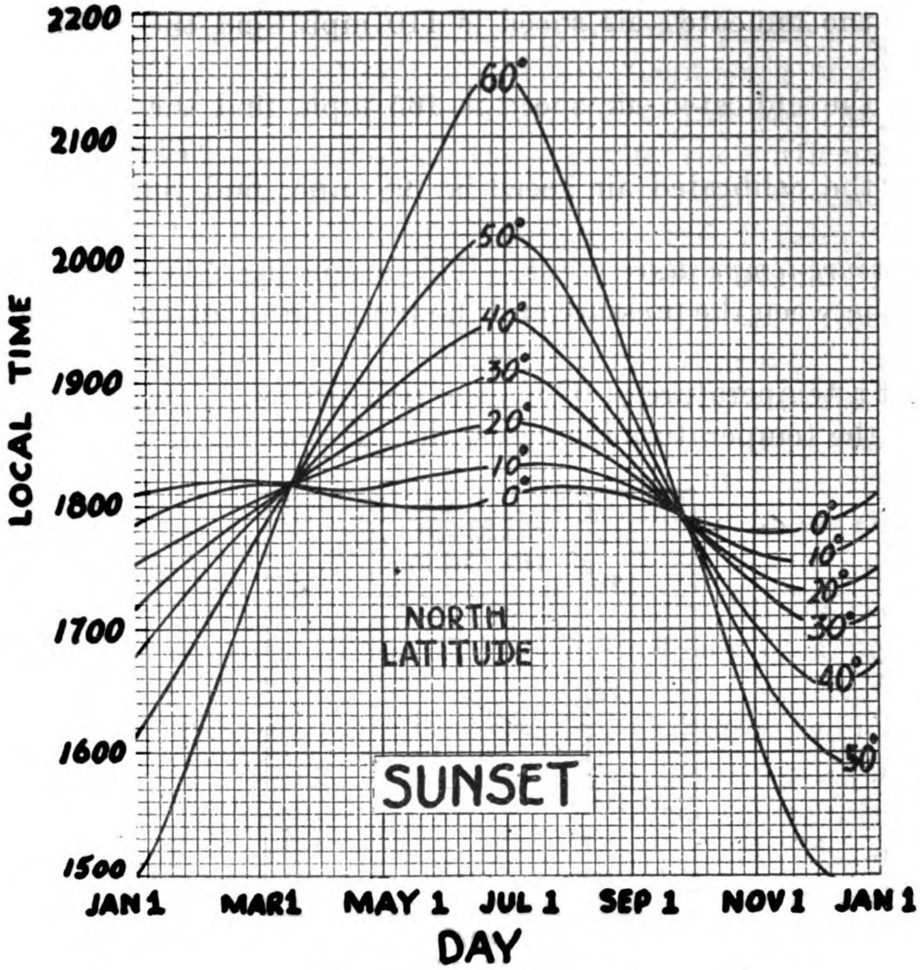


FIGURE 32.—Sunset graph.

d. Sunset graphs.—These graphs enable the pilot to determine the time of sunset for any position on the earth. The 24-hour system of keeping time is used in sunset graphs.

(1) *Instructions for use.*—(*a*) Enter the top or bottom scale with proper date.

(*b*) Move vertically down or up to the curve for observer's latitude (observer's position).

(*c*) Move horizontally to the right or left and read local civil time of sunset on vertical scales at the side.

(2) Find the sunset time for November 1 at latitude 30° N. (Follow instructions for use.) 1721 hr *Answer.*

(3) Find the sunset time for May 15 at latitude 50° N.

(4) Find the sunset time for May 20 at latitude 30° N.
1848 hr *Answer.*

(5) Find the sunset time for June 10 at latitude 10° N.

(6) Find the sunset time for February 10 at latitude 40° N.
1736 hr *Answer.*

(7) Find the sunset time for October 20 at latitude 30° N.

e. Fuel consumption graph.—This graph (fig. 33) shows the relationship between the air speed and the fuel consumption.

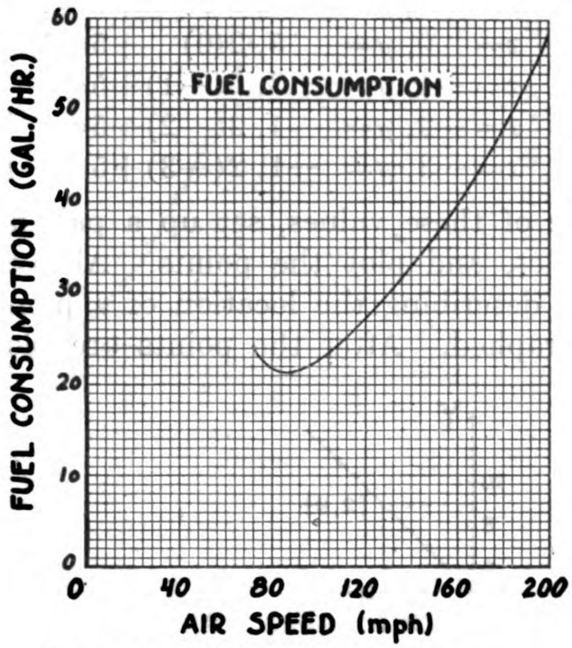


FIGURE 33.—Typical fuel consumption curve.

(1) At an air speed of 180 mph, the fuel consumption is ______ gallons/hour.

(2) At an air speed of 168 mph, the fuel consumption is ______ gallons/hour.

(3) If the fuel consumption is 53 gallons/hour, the air speed is ______ mph.

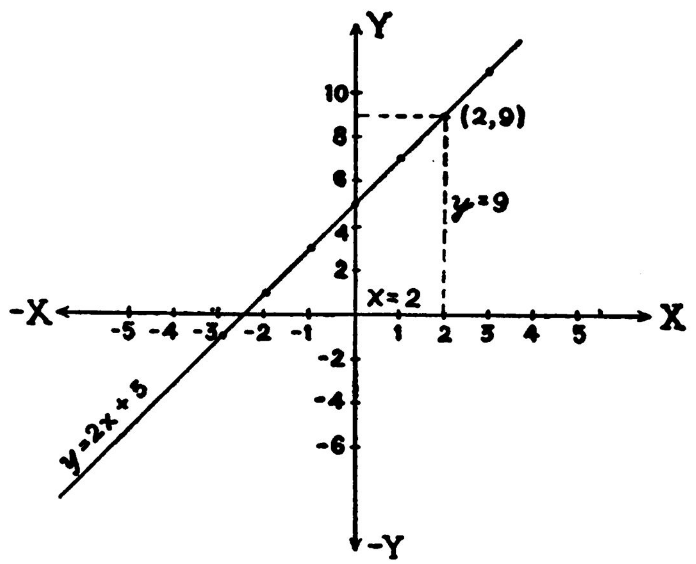
(4) If the fuel consumption is 39 gallons/hour, the air speed is ______ mph.

35. Examples of graphs.—*a.* The graphs that will be used and constructed will consist of a succession of points plotted with reference to coordinate axes and connected by a smooth line forming a straight line or a curve. The coordinates are obtained either from a formula or from empirical data (that obtained by observation).

b. Graphing from an equation.—(1) In algebraic work, various expressions suitable for graphing are employed, especially the equations. Consider the equation $y-2x=5$, or $y=2x+5$, and draw its graph. Since it is of the type $ax+by=c$ (a, b, c some known constants), the graph of which is always a straight line, the graph of $y=2x+5$ should be a straight line. If a number, for example 3, is substituted for x in the quantity $2x+5$, then the quantity $2x+5$ (or y) takes on the value 11. In a similar manner, many more pairs of numbers can be obtained which satisfy the above equation. For example:

$$
\begin{aligned}
\text{when } x&=3, & y&=11, & 2(3)+5&=11\\
x&=2, & y&=9, & 2(2)+5&=9\\
x&=1, & y&=7, & 2(1)+5&=7\\
x&=0, & y&=5, & 2(0)+5&=5\\
x&=-1, & y&=3, & 2(-1)+5&=3\\
x&=-2, & y&=1, & 2(-2)+5&=1\\
x&=-3, & y&=-1, & 2(-3)+5&=-1, \text{ etc.}
\end{aligned}
$$

Construct a table of these values, set up a pair of coordinate axes with suitable scales, and plot the points. Always remember that the two numbers describing the location of a point will satisfy the equation being graphed. After the points are plotted, notice that



x	(2x + 5) y
.	.
.	.
.	.
-3	-1
-2	1
-1	3
0	5
1	7
2	9
3	11
.	.
.	.
.	.

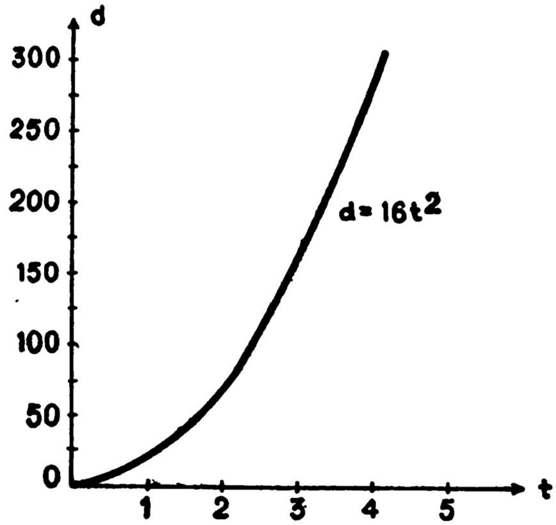
FIGURE 34.

they all lie in a straight line. Draw a straight line through the points. Every point on this line has a Y-value and an X-value satisfying the equation.

(2) A freely falling object will fall a distance d feet in t seconds as given by:

$$d=\frac{1}{2}gt^2$$

Use 32 for g, and the formula then becomes: $d=16t^2$. This formula is of a different type from those discussed above. After calculating a few values which satisfy this equation and drawing a graph representing the formula, figure 35 is obtained.



t	d
0	0
1	16
2	64
3	144
4	256
⋮	⋮

FIGURE 35.

Exercise: From the graph—

How far will an object fall in 2½ seconds?
Check by calculating this distance from the formula.

c. Exercises.—Plot the following points on a coordinate system of graph paper:

(1) (2, 3); (5, 7); (8, 0); (0, 5).

(2) (—2, 5); (—7, 8); (—5, 0); (—2, 3).

(3) (—2, —4); (0, —5); (—4, —7); (—2, —3).

(4) (1, —5); (2, —4); (5, —8); (2, —7).

(5) Three corners of a rectangle are at (1, 4), (4, 8), and (9, —2). Find the coordinates of the other corner.

(6) Compute a table of values and draw a graph of the following:

(*a*) $y=2x$; $y-3x=5$.

(*b*) $y=6$; $y-x=0$.

(*c*) $y-7=-2x$; $2y-3x=9$.

(*d*) $x=-3$; $\frac{y}{2}-\frac{x}{3}=1$.

(*e*) $y=\frac{1}{4}x^2$; $y^2=4x$.

(*f*) $d=5-t^2$.

(*g*) $2m=16+8t-t^2$.

(7) From the graph in figure 35, read off—

(*a*) The distance that an object will fall in 1½ seconds and in 3½ seconds.

(*b*) How long it will take an object to fall 50 feet? 100 feet? 225 feet?

(8) The velocity of sound in air depends on the temperature of the air. By use of the following data, draw a graph showing how the velocity varies with the temperature.

Velocity (ft per sec.)	1, 030	1, 040	1, 060	1, 080	1, 110	1, 140	1, 170
Temperature (F.)	—30°	—20°	0°	20°	50°	80°	110°

From the graph, find the velocity if the temperature is 35°; 10.5°; —25°; 120°.

1095; 1070.5; 1035; 1180 *Answers.*

(9) The effective disk area of a propeller depends on the diameter of the propeller. By use of the table below, draw a graph showing how the effective area A in square feet varies with the diameter in feet.

A (sq. ft)	4. 2	6. 5	9. 4	12. 7	16. 6	21
D (ft)	8	10	12	14	16	18

From the graph, find the area if the diameter is 9 feet; 12.5 feet; 14¾ feet.

36. Graphic solution of algebraic equations containing two unknowns.—*a*. It is sometimes necessary to find a pair of numbers that will satisfy two equations at the same time. One method of doing this is to graph each equation on one set of coordinate axes and find the intersection of the curves.

Example: Find the values of x and y which will satisfy the following two equations simultaneously:

$$2x-y=3$$
$$3x+2y=8$$

Solution: By graphing each equation separately on the same pair of coordinate axes, the line AB is obtained, every point of which has coordinates satisfying the equation $2x-y=3$; and the line CD is obtained, every point of which has coordinates satisfying the equation $3x+2y=8$.

The intersection of CD and AB is the point P the coordinates of which (2, 1) are the only ones satisfying both of the given equations.

b. This graphical method usually gives approximate results only, because of errors in measuring line segments when determining certain coordinates.

c. If the lines are parallel, it is obvious that no coordinate values will satisfy both of the given equations.

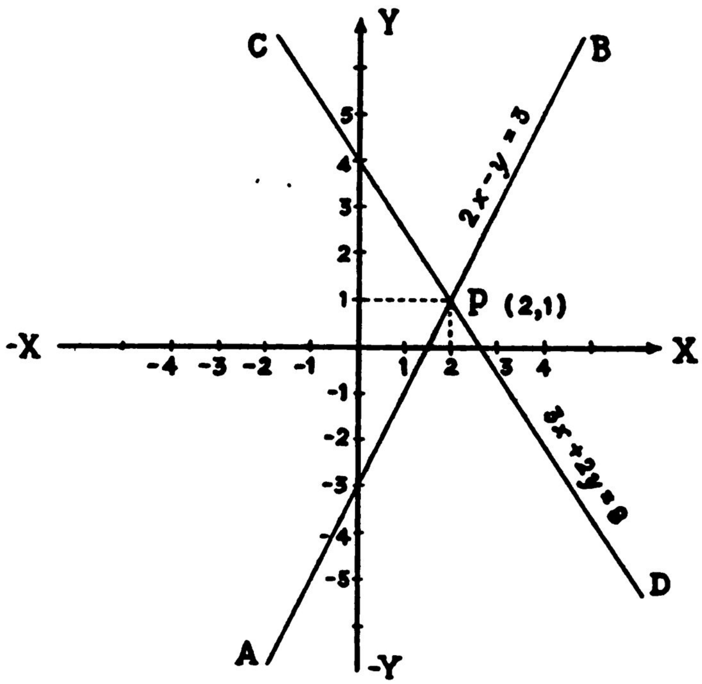


FIGURE 36.

d. Exercises.—Solve graphically—

(1) $x+y=3$
$x-2y=0$

(2) $2y-3x=0$
$4y+3x=-18$ (−2, −3) *Answer.*

(3) $x+2y=4$
$3x-y=6$

(4) $5y-3=0$
$10y+3x=4$ $(-\frac{2}{3}, \frac{3}{5})$ *Answer.*

e. Under certain circumstances, curves other than straight lines are plotted in pairs and their intersection found.

Example: Find graphically the values of x and y which satisfy the following two equations simultaneously:

$$2y=x^2$$
$$y+x=4$$

Solution: By graphing each equation separately on the same pair of coordinate axes, a curve is obtained, every point of which has coordinates satisfying the equation $2y=x^2$; and the line AB is obtained, every point of which has coordinates satisfying the equation $y+x=4$.

The intersections of the curve and the straight line are the points P and Q the coordinates of which (2, 2) and (−4, 8) are the only ones satisfying both of the given equations.

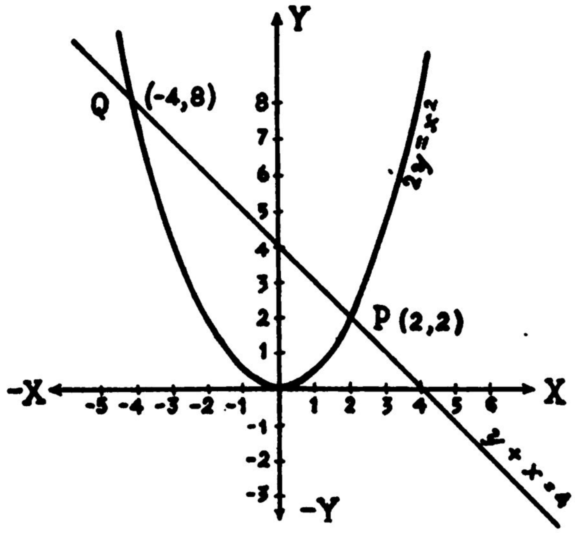


FIGURE 37.

f. Exercises.—Solve graphically—

(1) $y=x^2$
$y-x=0$

(2) $4y-x^2=0$
$y-x=0$ (0,0); (4,4) *Answer.*

(3) $y=x^2+2$
$y-x=4$

(4) $4y-15=x^2$
$x^2-3=y$ (3,6); (−3,6) *Answer.*

SECTION VI

ANGULAR MEASUREMENT

37. Purpose.—The purpose of this section is to familiarize the student airman with angular measurements and how they are used in determining directions in the U. S. Army Air Forces.

38. Angle and units of angular measurement.—*a.* Consider a circle. The circumference, or any part of that circumference which is called an arc, is divided into units called *degrees*. There are 360 degrees in a complete circumference (or one revolution). For more accurate measurements a degree is divided into 60 equal parts called *minutes* and a minute is divided into 60 equal parts called *seconds*.

The following table of angular measurement shows the symbols used for these units of angular measurement:

$$360^\circ \text{ (degrees)} = 1 \text{ circumference}$$

$$1^\circ = \frac{1}{360} \text{ part of a circumference}$$

$$60' \text{ (minutes)} = 1^\circ$$

$$60'' \text{ (seconds)} = 1'$$

Only the degree is used in this manual.

b. An angle is the figure formed by drawing two straight lines outward from a common point. The common point is called the vertex of the angle and the straight lines are called the sides of the angle. In figure 38, *O* is the vertex and *NO* and *PO* are the sides of the angle *NOP*. Another definition of an angle is the amount of rotation or turning necessary to rotate *NO* to the new position *PO*. The air navigation system of measuring and naming directions consists of designating directions by measuring them in degrees clockwise from the north through 360°. This angle is sometimes referred to as azimuth.

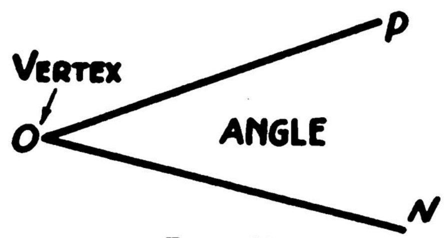


FIGURE 38.

c. The instrument for measuring angles is the protractor. To measure an angle with a protractor, place the protractor on the angle to be measured (see fig. 39) so that either half of the side *AB* will fall upon one side of the angle and the point *O* on the vertex. The reading on the scale where the other side of the angle crosses it is the measure of the angle in degrees.

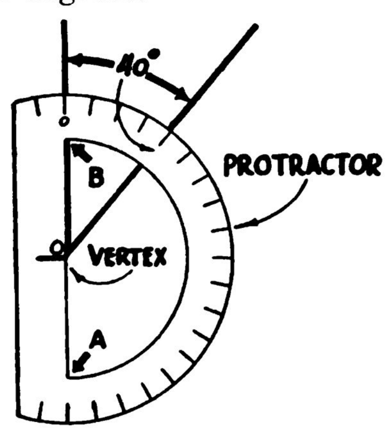


FIGURE 39.

d. *Exercise:* Measure each of the angles in figures 40, 41, 42, and 43.

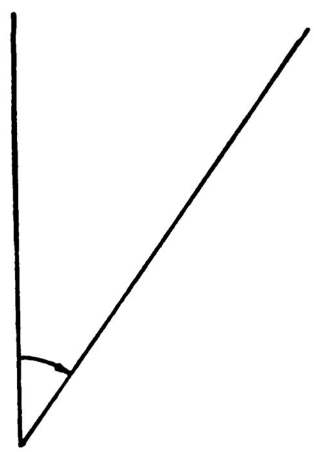


FIGURE 40.

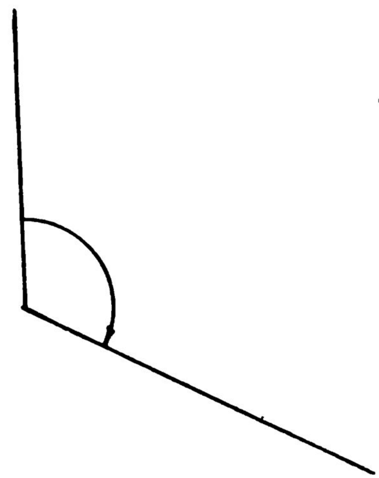


FIGURE 41.

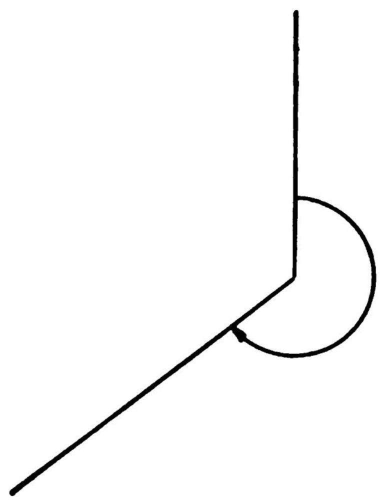


FIGURE 42.

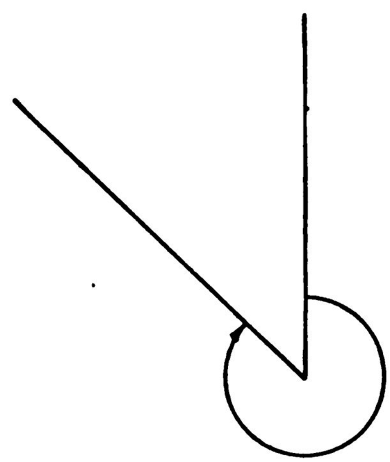


FIGURE 43.

e. *To construct an angle with a protractor.*—Draw one side of the angle and locate the vertex. Place side *AB* of the protractor on the side drawn with point *O* of the protractor on the vertex. Locate the reading of the value of the angle required on the scale of the protractor and connect this with the vertex. Measure the angle by starting at the north line.

(1) *Example:* Lay off an angle of 115°.

Solution: First draw the north line, then follow the instructions in *c* above (see fig. 44).

(2) *Exercise:* Use a protractor and lay off angles of 30°, 135°, 180°, 240°, and 315°.

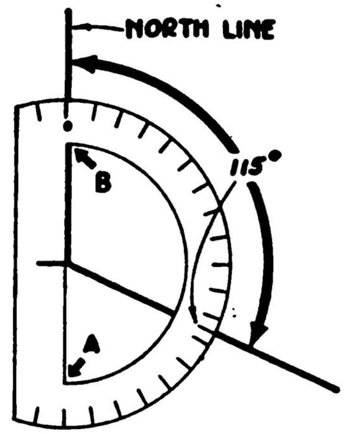


FIGURE 44.

f. Angles of elevation and depression.—The angle of elevation or angle of depression of any given object is the angle made by the line to that object and a horizontal line at the eye level in the same vertical plane. In figure 45 the angle made with the two airplanes is the angle of elevation while the angle made with the power lines is the angle of depression.

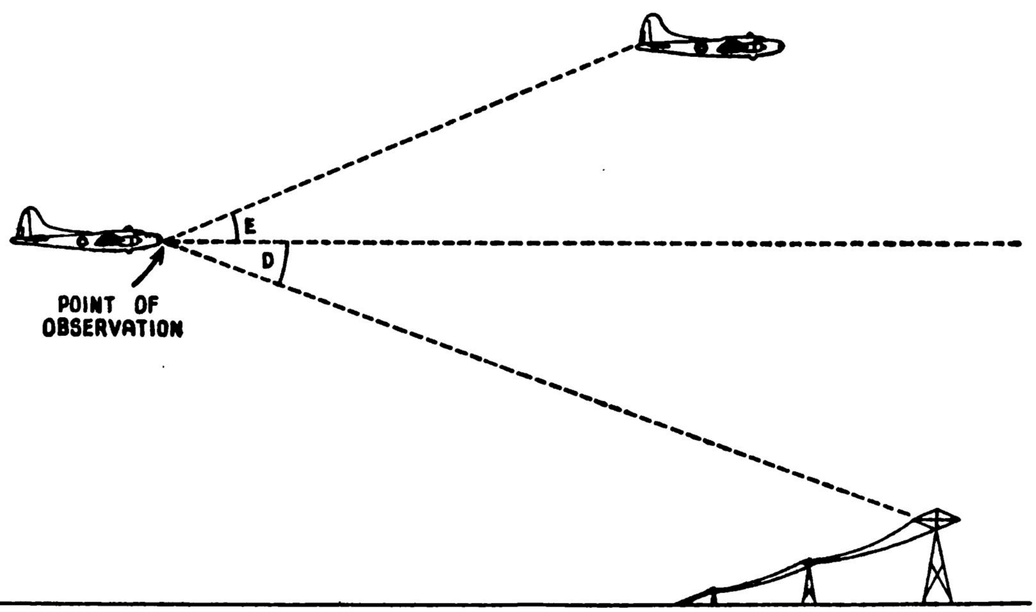


FIGURE 45.

39. Course, heading, drift.—*a. True course* (C)—Direction over the surface of the earth expressed as an angle with respect to true north in which an aircraft intends to fly. It is the direction as laid out on a map or chart.

b. True heading (H).—Angular direction of the longitudinal (front to rear) axis of the aircraft with respect to true north.

c. Drift (D)—Angle between the true heading and the true course. It is *right* drift if the true course is *greater* than the true heading. If the true course is *less* than the true heading, it is *left* drift.

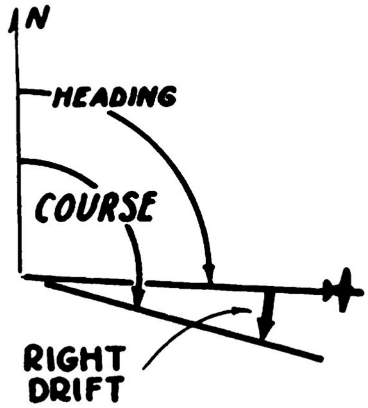


FIGURE 46.

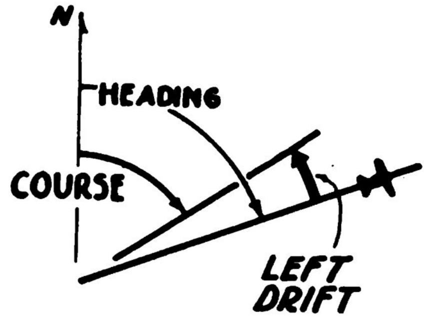


FIGURE 47.

d. Wind direction.—Wind is designated by the direction from which it blows (see fig. 48).

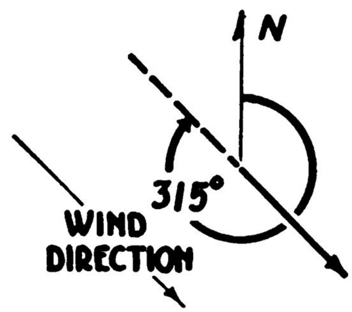


FIGURE 48.

e. In determining true heading when true course is given, subtract right drift from true course. Add left drift to true course to obtain true heading.

40. Miscellaneous exercises.—*a.* Determine the direction (north, south, east, west, etc.) in each of the following cases:

(1) True course 180°.
(2) True course 270°. True course west. *Answer.*
(3) True course 225°.
(4) Wind from 315°. Wind from northwest. *Answer.*
(5) Wind from 135°.
(6) Wind from 0°. Wind from north. *Answer.*
(7) True heading 45°.
(8) True heading 90°. True heading east. *Answer.*

b. Determine the true heading in each of the following cases:

(1) True course = 90°, right drift =6°.

Solution:

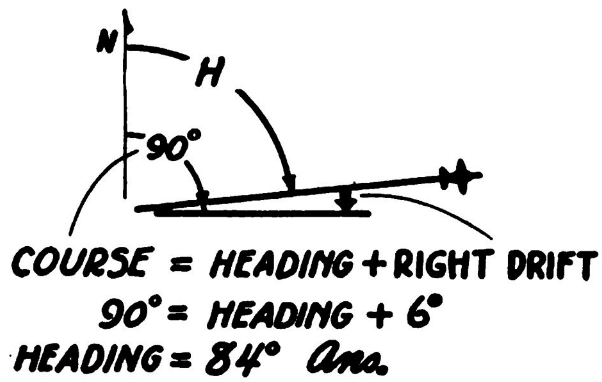


FIGURE 49

(2) True course = 135°, left drift = 9°. True heading = 144° *Answer.*

(3) True course = 270°, left drift = 11°.

(4) True course = 315°, right drift = 10°. True heading = 305° *Answer.*

(5) True course = 0°, left drift = 15°.

SECTION VII

VECTORS

41. Purpose.—The purpose of this section is to give the student airman an idea of what a vector is and of how vectors are used in triangle of velocity problems.

42. Scalars and vectors.—*a.* A scalar is a quantity which has magnitude only. Such quantities can be completely specified by number or letter. Whenever one number completely specifies a quantity, that quantity is a scalar: distance, time, and speed are scalars, as in 6 feet, 30°, 40 mph, or *y* miles.

b. A vector is a quantity which has direction as well as magnitude. Thus two numbers must be given to determine a vector, one for magnitude and one for direction. An automobile traveling north at 10 feet/second needs a direction as well as a magnitude to describe its motion, so its velocity (speed and direction) is a vector quantity.

c. Representation of a vector.—Although a vector is determined by two quantities, it can be represented graphically by an arrow having a length which depends on the magnitude of the vector and a direction which is the given direction.

(1) To find the proper length for a vector, a scale must be chosen arbitrarily, and some appropriate length must be selected to represent the given magnitude. A scale should be chosen which will give the best sized diagram. Large diagrams give more accurate results. For example, if 1 inch represents 100 pounds of force, a line 2¼ inches long would stand for a force of 225 pounds.

(2) The direction is measured clockwise from the north.

43. Addition of vectors.—*a.* The sum of two vectors, V_1 and V_2 is a third vector which is written $V_3=V_1+V_2$. This is called the *resultant* of V_1 and V_2.

b. The resultant, when applied alone, gives the same result obtained by applying the two original vectors separately.

(1) To add two vectors, V_1 and V_2, graphically, draw both from a point O, then complete the parallelogram which has V_1 and V_2 as two of its sides. An arrow from O to the opposite vertex of the parallelogram is the resultant.

(2) *Example:* To add the vectors V_1 and V_2:

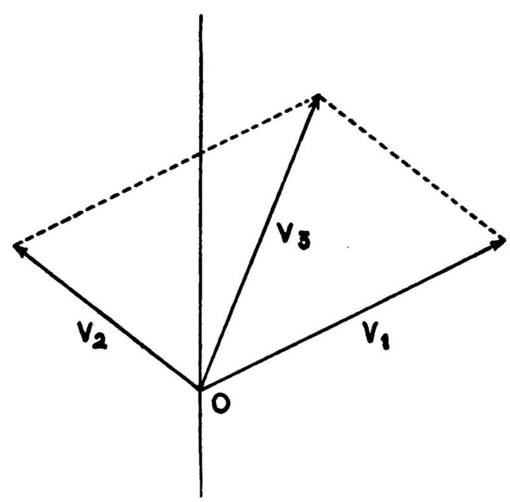


FIGURE 50.

(V_3 is the sum or resultant of V_1 and V_2.)

c. An alternate method of adding vectors is drawing $V_1=OA$ from any point, O, and $V_2=AB$ from the arrow end of V_1, with arrow away from arrow end of V_1. Then join O and B by an arrow. $V_3=OB$ is the resultant. This method may be used to add any number of vectors. Merely add each vector to the preceding vector. The resultant is obtained by joining the point O to the arrow of the last vector.

(1) *Example:* To add the vectors V_1 and V_2:

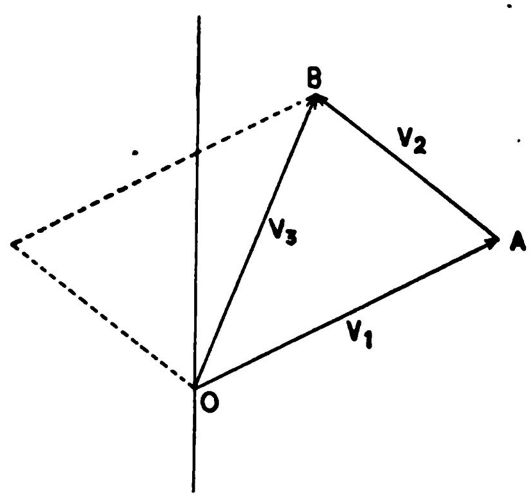


FIGURE 51.

The dotted lines are unnecessary but are added here to show the similarity to the previous method.

(2) *Example:* To add the vectors V_1, V_2, V_3, and V_4:

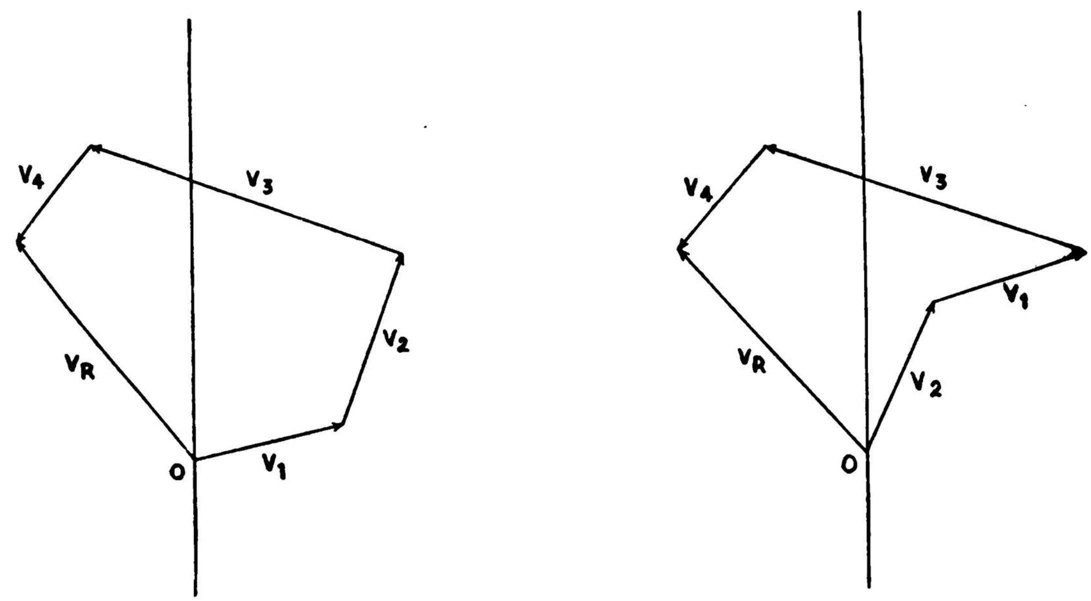


FIGURE 52.

NOTE.—The vectors may be added in any order without changing the resultant as long as the direction and magnitude of each vector are preserved.

d. Exercises:

(1) Tell which of the following are vectors and which are scalars:

(*a*) 2 pounds.
(*b*) 20 mph east.
(*c*) 10 feet/second.
(*d*) Acceleration of a falling body.
(*e*) A velocity.

(2) Graph the following:

(*a*) 30 mph NW.
(*b*) 40 yards/minute north.
(*c*) 30 pounds force acting N 10° E.
(*d*) 17 pounds force acting S 50° W.
(*e*) 99 pounds at 182°.
(*f*) 14 mph at 37°.
(*g*) 18 feet/second at 282°.

(3) Add the following vectors ($V_1 + V_2$):

(*a*) V_1: Length =340, Azimuth=323° — V_2: Length =410, Azimuth=20°

(*b*) V_1: Length =10, Azimuth=200° — V_2: Length =15, Azimuth=140°

Length=22, azimuth=163° *Answer.*

(*c*) V_1: Length =.05, Azimuth=270° — V_2: Length =.16, Azimuth=10°

(*d*) V_1: Length =12, Azimuth=82° — V_2: Length =18, Azimuth=126°

Length=28, azimuth=108° *Answer.*

(*e*) V_1: Length =163, Azimuth=191° — V_2: Length =313, Azimuth=264°

(*f*) V_1: Length =.5, Azimuth=0° — V_2: Length =.73, Azimuth=180°

Length=23, azimuth=180° *Answer.*

(4) A car moves at the rate of 40 mph due north for 2 hours, then turns and goes due east for 1 hour. Find by vector diagram how far in a straight line the car is from its starting point. How much time would have been saved by traveling the straight route?

89.44; .764 hr or 45.8 min. *Answer.*

(5) A body is subjected to a force of 50 pounds acting toward 50° and a force of 70 pounds acting toward 340° at the same time. Find the direction and magnitude of a third force that will counterbalance exactly the effect of the first two.

(6) A man heads the bow of his boat directly across a river which is flowing at the rate of 2 mph. If he rows r mph, how much off his course is he at the end of 1 hour? How far does he go in 1 hour?

2 miles; $\sqrt{r^2+4}$ *Answer.*

(7) A man pushes on the handle of a lawn mower with a force of 75 pounds in line with the handle. The handle makes an angle of 30° with the horizontal. Find how much force is—

(*a*) Straight ahead.

(*b*) Into the ground.

44. Triangle of velocity.—*a.* The velocity of an object may be defined as a vector quantity the magnitude of which is the speed of the object and the direction of which is the direction of the object's motion. It can be represented by a directed line segment (vector). The velocity of a body which is acted upon by two forces can be found by considering the velocity resulting from each force individually. The actual, or total, velocity of the body is the resultant of the separate velocities. The velocity, relative to the ground, of an airplane in flight is the resultant of two velocity vectors: the air speed and heading vector, and the wind velocity vector. This resultant velocity vector is the course and ground speed vector.

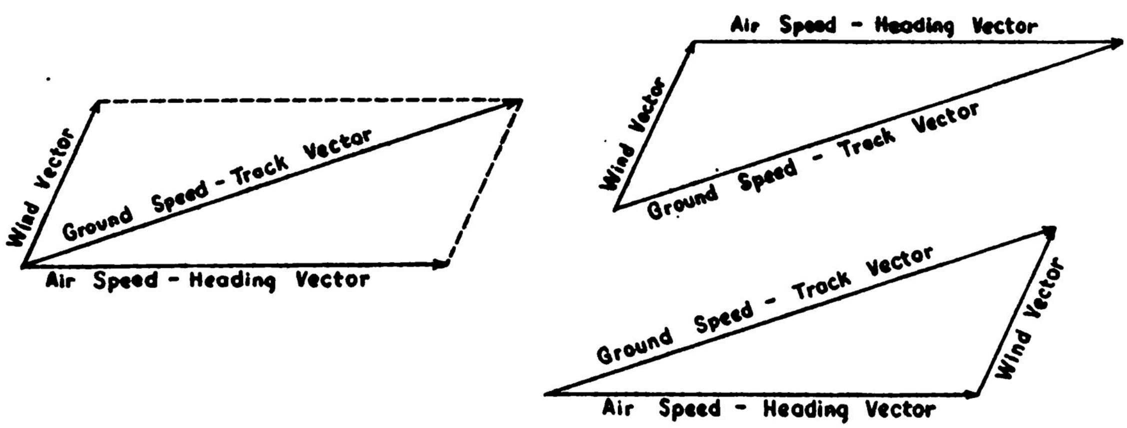


FIGURE 53.

b. The terms used are now defined as follows:

(1) *Air speed.*—The true speed of an aircraft relative to the air. Speed is given in mph or knots.

(2) *True heading.*—The angular direction of the longitudinal axis of the aircraft with respect to true north. The words "heading" and "true heading" will be used interchangeably.

(3) *Wind direction and speed.*—Wind is designated by the direction *from which it blows.* Its speed is expressed in mph or knots.

(4) *Ground speed.*—The actual speed of an aircraft relative to the earth's surface in mph or knots.

(5) *True course.*—The direction over the surface of the earth, expressed as an angle, with respect to true north, that an aircraft is intended to be flown. It is the course laid out on the chart or map. Strictly, *track* is the actual path of an aircraft over the surface of the earth *that has been flown.* In practice, "track" and "course" are freely used interchangeably.

c. Some general rules for setting up vector diagrams follow:

(1) The reference system for direction of vectors is always the same. The true north line is the reference line and all angles are measured from it clockwise, through 360°.

(2) Choose a scale which will give lines *large enough* for accurate measurement.

(3) Label all vectors as they are drawn.

(4) "Heading" and "air speed" determine one vector.

(5) "Course" and "ground speed" determine one vector.

(6) *Wind always blows* the aircraft from the heading *onto the course.*

45. Type I.—The simplest triangle of velocity problem involves finding the *true course* and *ground speed* when wind speed, wind direction, air speed, and heading are known.

a. Example: The pilot has been told to fly a heading of 56° in an airplane the air speed of which is 100 mph. The wind is blowing 25 mph from 110°. Find the true course and ground speed.

NOTE.—See back of manual for figure 54.

(1) *Parallelogram method.*—To find the true course and ground speed, add the wind velocity vector and the air speed heading vector by the parallelogram method. The length of the resultant determines the ground speed and the direction of the resultant is the true course direction. The length of the resultant is 8.7 cm, thus the ground speed is 87 mph. The direction is 42°; thus, true course equals 42°.

(2) *Triangle method.*—To find the true course and ground speed, add the wind velocity vector and the air speed heading vector by the triangle method.

(*a*) Draw the north line.

(*b*) Draw the air speed heading vector at an angle of 56° from the north, making it 10 cm long (1 cm=10 mph; hence 10 cm=100 mph).

(*c*) Draw the wind vector away from 110°, making it 2½ cm long (2½ cm=25 mph) and from the head of the air speed heading vector.

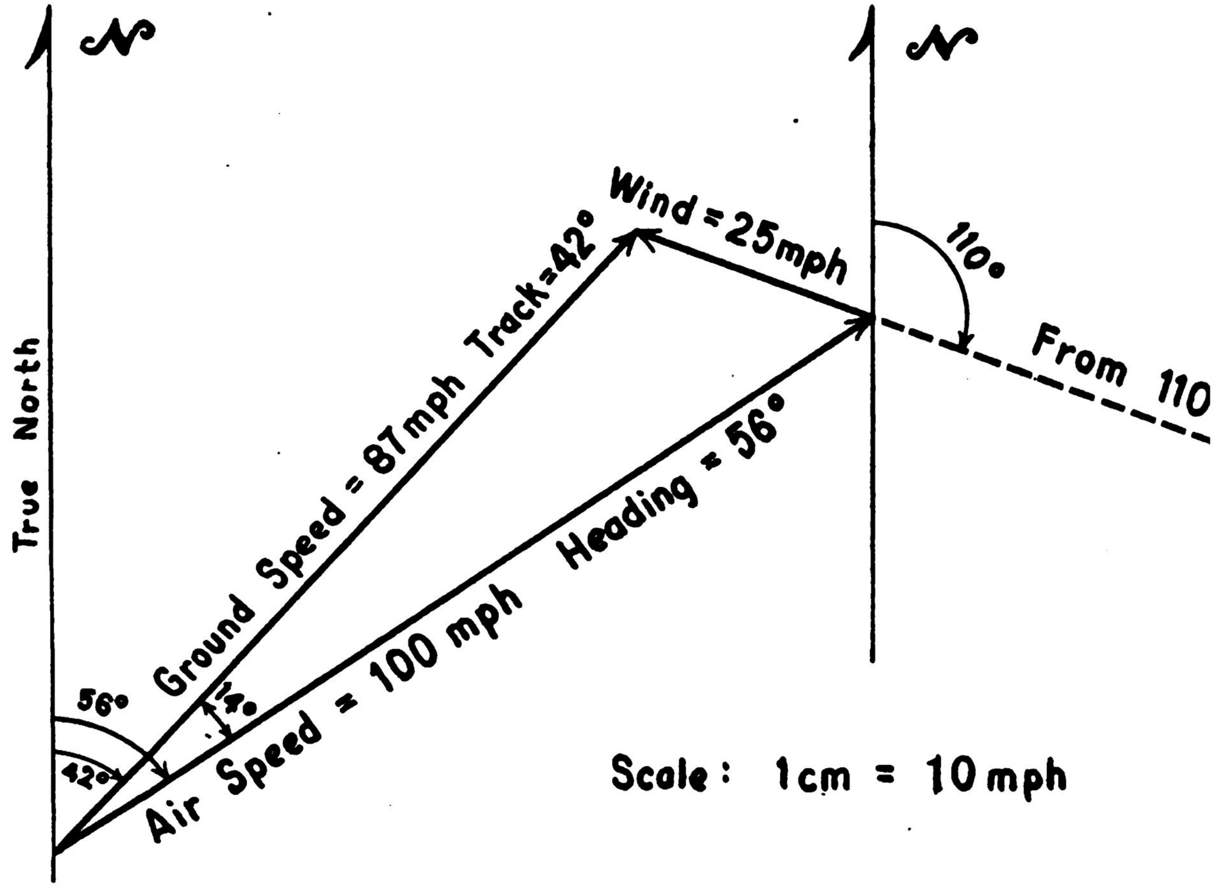


FIGURE 55.

(*d*) Air speed heading vector+wind vector=ground speed track vector ($V_a + V_w = V_g$).

(*e*) The angle between the ground speed track vector and the north line determines the course or track. The length of the ground speed track vector, according to the scale used, determines the ground speed.

(*f*) The drift angle is the angle between the air speed heading vector and the ground speed track vector. Since this is a case of left drift, the drift angle of 14° is subtracted from the heading of 56° to obtain the course 42°.

b. Exercises.—(1) On a secret maneuver, the pilot has been ordered to fly on a heading of 90° until further orders. Air speed of the airplane is 180 mph. After 1 hour of flight, motor trouble develops and a landing must be made. He is told over his radio that wind has been 40 mph from 135°. What has been his track and ground speed?

(2) The pilot observes an enemy scouting party. To avoid being seen, he changes his heading (as point "A") and flies on a heading of 315° with air speed 150 mph. If wind is 30 mph from 45°, what will be his new course and ground speed? C=304°, GS=153 *Answer.*

(3) Because of an error in plotting his course, a pilot finds himself over unfamiliar territory. His air speed has been 160 mph, his

heading has been 180°, and he verifies that wind throughout his flight has been 20 mph from 315°. What has been his track and ground speed?

(4) Heading has been 30° and air speed of plane is 180 mph. If wind is 60 mph from 240°, what has been the course (or track) and ground speed? C=37°, GS=234 *Answer.*

(5) Figure 54 was carefully drawn, but was not properly labeled. Give two different and equally reasonable sets of facts which it might be intended to represent.

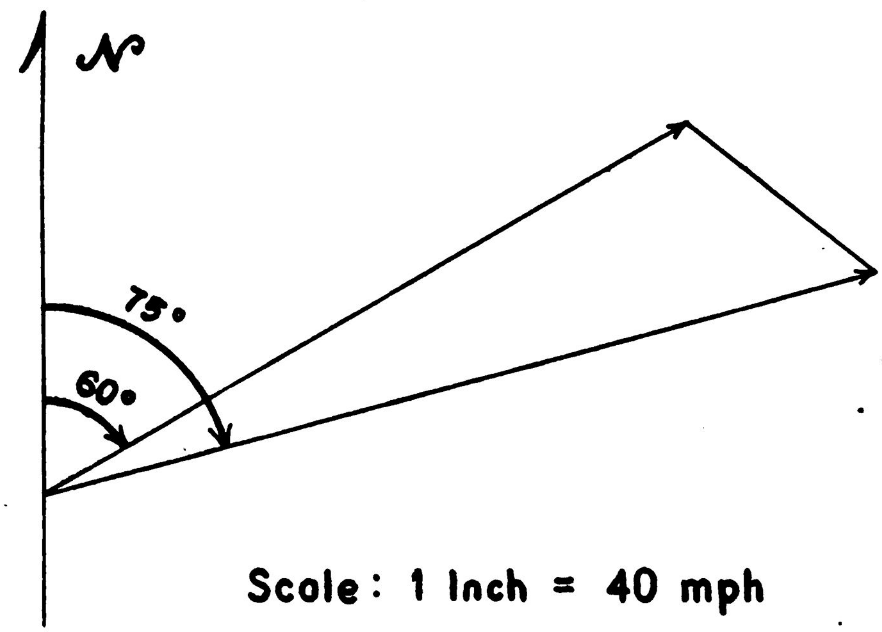


FIGURE 56.

46. Type II.—The type of problem next to be considered is one that must be worked out before the flight is started. As a flight is being planned, a certain course is desired. A line showing the desired course is laid off on the map and its azimuth (course angle) is measured. Wind direction and speed are accurately observed and reported at frequent intervals. The air speed of the airplane to be used is known. The heading which the airplane must have in flight in order to make good the desired course, may be determined. Heading and ground speed over the actual course may be read from a vector diagram.

a. Example.—Course to be flown is 308°. Air speed of the airplane is 110 mph and wind velocity is 15 mph from 230°. Find the heading and ground speed.

Note.—See back of manual for figure 57.

b. Solution: First, the desired course is indicated by a line of indefinite length drawn in the direction 308°. Since the parallelogram method is to be used to find the unknown parts of the vector diagram, the wind velocity vector originates at the same point as the course line. Wind velocity, 15 mph, is represented by a line 1.5 cm long from the direction 230°. Since air speed, 110 mph, is known but heading is not, the compass is set with a radius of 11 cm (11 cm=110 mph), and, using the head of the wind velocity vector as center, strike an arc which will intersect the course line. The line drawn from this point to the head of the wind vector has the length and direction of the air speed heading vector. The segment of the course line cut off by the intersecting arc is the length representing the ground speed. The parallelogram is completed in such a way that all three vectors originate from the origin point with the wind velocity vector and the air speed heading vector as sides and the ground speed track vector as the diagonal.

c. The solution of the same problem by the triangle method is given next.

(1) Draw the north line.

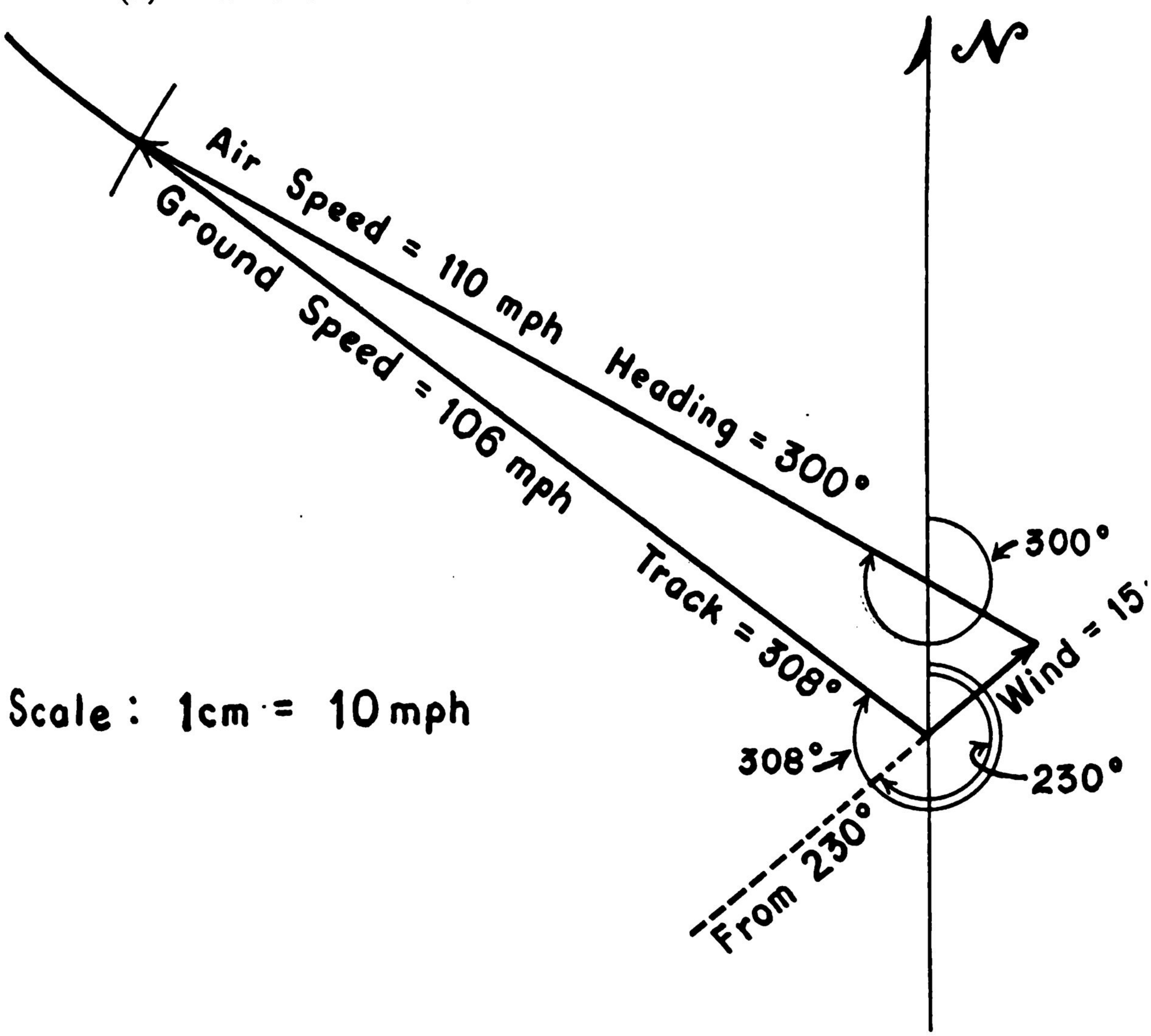


FIGURE 58.

(2) Draw the wind velocity vector from 230°, making it 1.5 cm long.

(3) Draw a line of indefinite length in the direction 308°, indicating the desired course.

(4) Draw the air speed heading vector by placing one end of a ruler on the head of the wind vector and mark where a vector 11 cm long will touch the course line.

(5) The segment of the course line cut off by the head of the air speed heading vector represents the ground speed.

(6) The angle between the air speed heading vector and the north line measured clockwise determines the heading.

d. Exercises.—(1) Course to be flown is 270°. Air speed of the airplane is 120 mph, and wind is 40 mph from 45°. Find the required heading and the ground speed.

(2) Wind is 30 mph from 180°. Desired course is 45°, and air speed of the airplane is 140 mph. What should be the heading, and what will be the ground speed?

H=54°, GS=160 *Answer.*

(3) Air speed of the airplane is 125 mph. What will be the required heading to fly a course 135° if wind is blowing 25 mph from 225°? What will be the ground speed?

(4) Course to be flown is 225°, and wind is 30 mph from 90°. If air speed of the airplane is 160 mph, find the required heading and the ground speed. H=217°, GS=180 *Answer.*

(5) Wind is 45 mph from 10°, and air speed of the airplane is 165 mph. If the course to be flown is 150°, what will be the required heading and what will be the ground speed?

47. Type III.—If the air speed, heading, ground speed, and track (angle of actual course flown) are known, the wind velocity can be found. These cases arise when the first two factors are obtained from instrument reading and the last two are computed by timing the flight between two landmarks on a map.

a. Plot both the air speed heading vector and the ground speed track vector from the given data.

b. Draw a vector with tail at the arrow end of the air vector and arrow at the arrow end of the ground vector. This is the wind vector.

c. By drawing a north line through the tail of the wind vector, one can measure the azimuth of the wind vector, that is, get the wind direction. The length represents the wind speed.

d. Note that the sum of the wind vector and the air vector is the ground vector; that is, $V_W + V_A = V_G$.

e. Example: An airplane's air speed is 155 mph and its heading is 240°. By computation from a chart, it is found that the ground speed is 170 mph and the track is 251°. What is the wind velocity?

From the vector diagram the wind velocity is 33 mph from 129° *Answer.*

Note.—See back of manual for figure 59.

48. Summary.—In any vector triangle, there are six quantities involved: the length and the direction of each of the three vectors.

a. It can be seen from the preceding examples that when *any* four of the six quantities are given, the other two can *always* be found. However, the triangle cannot be solved if less than four of the quantities are given.

b. The two unknown quantities can always be found by using $V_W + V_A = V_G$, or briefly *W A G.*

c. Exercises.—(1) If an airplane with an air speed of 180 mph flies with a heading of 160° in a 20-mph wind blowing from 215°, what are its track and ground speed?

(2) If an airplane must fly a 600-mile course, which is 350° in 4 hours, while a 40-mph wind is blowing from 25°, what must its air speed be? AS=184, H=357° *Answer.*

(3) If an airplane with an air speed of 310 mph must fly a 200° course in a 35-mph wind blowing from 180°, what must its heading be, and what will its ground speed be?

(4) An airplane's air speed is 160 mph, its heading 10°. The wind velocity is 28 mph from 116°. What are the track and ground speed? GS=170, T=1° *Answer.*

(5) An airplane with an air speed of 130 mph must fly a true course of 127° in a 25-mph wind from 221°. What must be the heading, and what will the ground speed be?

(6) An airplane with an air speed of 150 mph and a heading of 65° flies a course of 80° with a ground speed of 140 mph. Find the wind velocity. WS=40, WD=357° *Answer.*

(7) An airplane's air speed indicator reads 205 mph and its compass 67°. The navigator notes that the airplane passes over two landmarks which are given on his map. The distance between them is 310 miles and the direction of the line between them is 76°. If the airplane took 1 hour, 40 minutes to fly from one to another, what is the wind velocity?

(8) An airplane with an air speed of 245 mph must fly a course of 310° and return to the same base along the same line. A 280° wind is blowing 28 mph. How long should the pilot fly out along the line if he has 3 hours' fuel? What heading must he take in and out?

T=1 hr 39 min.; H in=133°; H out=307° *Answer.*

(9) A pilot flying an airplane which has an air speed of 200 mph takes off from field *A* to fly to field *B* which lies 800 miles and 315° from field *A*. A wind of 30 mph is blowing from 0°. The pilot miscalculates his course and lands at field *C* to get his bearing. *C* is 565 miles from both *A* and *B*, and is due north of *A*, and due east from *B*. How much time did he lose by not flying a direct line?

49. Miscellaneous exercises.—(1) If an airplane with an air speed of 160 mph and a heading of 312° flies in a 35-mph wind from 20°, what are the track and ground speed?

(2) An airplane with an air speed of 135 mph must fly a course of 93° in a 30-mph wind from 225°. What must be the heading, and what what will the ground speed be? H=102°; GS=153 *Answer.*

(3) An airplane's heading is 35°. A 25-mph wind is blowing 95°. If the airplane's air speed is 120 mph, what are its ground speed and track?

(4) An airplane's air speed is 210 mph. A 45-mph wind is blowing 195°. If the airplane must fly a course of 248°, what must the heading be and what will the ground speed be? H=238°; GS=180 *Answer.*

(5) An airplane must fly a 75° course and maintain a ground speed of 165 mph. A 40-mph wind is blowing from 160°. What must the air speed and heading be?

(6) In a 30-mph wind from 350°, an airplane flies with a heading of 235° and an air speed of 170 mph. What are the track and ground speed? T=226°; GS=185 *Answer.*

(7) An airplane must fly a 23° course in a 25-mph wind from 272°. If the airplane's air speed is 140 mph, what must the heading be? What will the ground speed be?

(8) An airplane must fly from A to B in 4 hours 30 minutes. These fields are 630 miles apart and the direction of the line between them is 210°. If the wind is 32 mph from 105°, what must the heading and air speed be? H=196°; AS=136 *Answer.*

(9) An airplane with an air speed of 135 mph and a heading of 355° passes over a landmark, and 1 hour 20 minutes later passes over another which is 200 miles and 10° from the other. What is the wind velocity?

(10) An airplane with an air speed of 170 mph and a heading of 177° flies directly over a road the direction of which is 165°. The pilot sees smoke blowing from a chimney from 275°. What are the ground speed and wind speed? GS=178; WS=37 *Answer.*

(11) An airplane with an air speed of 190 mph must fly along a course of 310° and return along the same line. Total flying time is

to be 2 hours 30 minutes. If the wind is 40 mph from 190°, how long should the pilot fly out along the line before turning back?

(12) A pilot flying an airplane with an air speed of 175 mph must land at either field *A* or field *B* as soon as possible. Field *A* is 117 miles and 30° from the airplane, while field *B* is 141 miles and 195° from the airplane. If the wind velocity is 40 mph from 335°, to which field should the pilot fly? Arrives at *B* 6 minutes sooner. *Answer.*

(13) An airplane with an air speed of 150 takes off from a field and flies with a heading of 305°. A 35-mph wind is blowing from 200°. Forty minutes later an airplane with an air speed of 190 mph takes off from the same field to overtake the first airplane. What heading must the pilot fly, and how long will it take him to overtake the other airplane?

APPENDIX

MISCELLANEOUS UNITS AND CONVERSION FACTORS

1. Formulas.

a. Area of circle, radius $= r$, is $\pi r^2 (\pi = 3.1416)$.

b. Area of rectangle, width $= w$, and length $= l$, is wl.

c. Area of triangle, base $= b$, and altitude $= h$, is $bh/2$.

d. Volume of sphere, radius $= r$, is $\frac{4}{3} \pi r^3 (\pi = 3.1416)$.

e. Volume of prism, area of base $= A$, height $= h$, is Ah.

2. Conversion factors.

66 nautical miles	=76 statute miles=122 kilometers (approximate)
1 centimeter (cm)	=0.393700 inch (in.)
1 foot (ft)	=12 inches (in.)
	=30.4801 centimeters (cm)
1 inch (in.)	=2.54001 centimeters (cm)
1 kilometer (km)	=1,000 meters (m)
	=0.62137 statute mile (mile, stat.)
	=0.53959 nautical mile (mile, naut.)
	=3280.83 feet (ft)
1 meter (m)	=100 centimeters (cm)
	=1,000 millimeters (mm)
	=3.28083 feet (ft)
	=39.3700 inches (in.)
1 millimeter (mm)	=0.039370 inch (in.)
1 nautical mile (naut.)	=6,080.20 feet (ft)
	=1.151553 statute miles (miles, stat.)
	=1.853249 kilometers (km)
1 statute mile	=5,280 feet (ft)
	=1.60935 kilometers (km)
	=.868393 nautical mile
1 U. S. gallon (gal.)	=231 cubic inches (cu. in.)
	=.13368 cubic foot (cu. ft)
	=.83310 British gallon
1 cubic foot (cu. ft)	=1,728 cubic inches (cu. in.)
	=7.4805 United States gallons (gal.)
1 kilometer per hour (km/hr)	=0.62137 statute mile per hour
	=0.53959 knot

1 knot	= 1 nautical mile per hour
	= 1.853249 km/hr
	= 1.151553 mph or miles/hr
1 statute mile per hour (mph or mile/hr)	= 1.4666 feet per second
	= 1.60935 km/hr
	= 0.868393 knot

3. Temperature scales.

Freezing: 0° C. = 32° F. = 273° K. (Absolute)

Boiling: 100° C. = 212° F. = 373° K. (Absolute)

$$C° = \frac{5}{9}(F° - 32).$$

$$F° = \frac{9}{5}C° + 32.$$

$$K° = C° + 273.$$

4. Time equivalents.

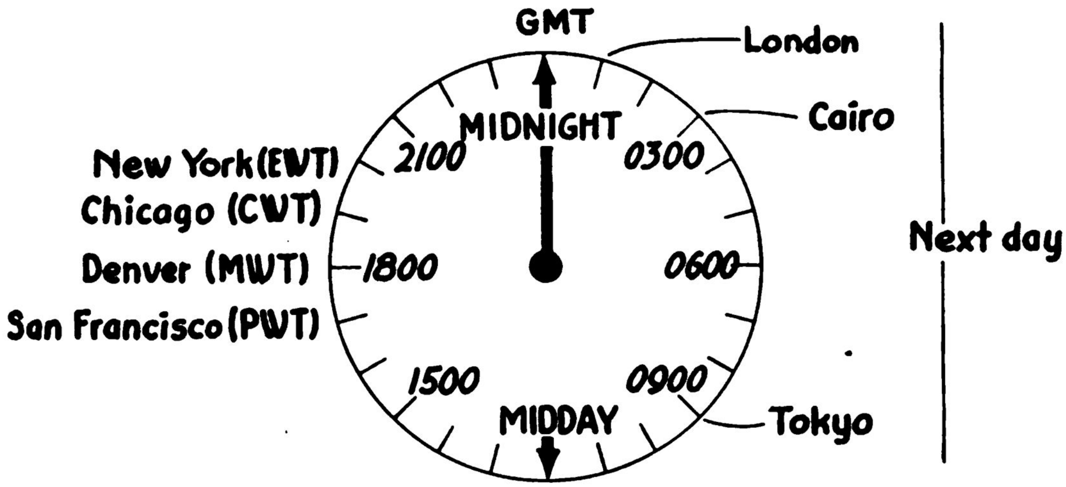


FIGURE 60

NOTE.—Time given for Tokyo is standard time. Difference in standard time between New York and Tokyo is 14 hours.

5. Densities.

Gasoline (aviation) weighs 45 lb/cu. ft or 6 lb/gal.

Oil (aviation) weighs 56 lb/cu. ft or 7.5 lb/gal.

Water weighs 62.4 lb/cu. ft or 8.34 lb/gal.

Air (dry) weighs .0765 lb/cu. ft at 15° C. (59° F.) and standard atmospheric pressure.

INDEX

INDEX

[A. G. 062.11 (1-11-43).]

BY ORDER OF THE SECRETARY OF WAR:

G. C. MARSHALL,
Chief of Staff.

OFFICIAL:

J. A. ULIO,
Major General,
The Adjutant General.

DISTRIBUTION:

R and H 1 (6); 44, 4 (5); IBn 1, 4, 44 (10).
(For explanation of symbols see FM 21-6.)

U. S. GOVERNMENT PRINTING OFFICE: 1943

For sale by the Superintendent of Documents, U. S. Government Printing Office
Washington, D. C.


mph
ON